Conceptual Article

Dynamic Geometry Enriches the Design of Curriculum Materials for Middle Secondary School Mathematics

Dr. Robyn Pierce and Dr. Kaye Stacey

Abstract

This paper examines the role of Dynamic Geometry Software (DGS) in enriching the design of curriculum materials for middle secondary school mathematics. It highlights how interactive visualisation, construction, and manipulation of geometric figures foster students' understanding of key mathematical concepts. The study emphasizes how DGS supports conjecture, exploration, and reasoning, moving beyond rote learning to deeper conceptual engagement. Design principles for integrating dynamic geometry into curriculum tasks are discussed, with a focus on enhancing student motivation, spatial reasoning, and problem-solving abilities. The authors argue that well-structured curriculum materials incorporating dynamic geometry provide valuable opportunities for both students and teachers to experience mathematics as an active, exploratory discipline.

Keywords: *Dynamic Geometry Software (DGS), Curriculum Design, Middle Secondary Mathematics, Geometric Reasoning, Visualization, Conjecture and Proof Mathematics Education*

INTRODUCTION

The RITEMATHS project explored the use of technology to support the use of real world contexts to enhance the learning of mathematics in the middle secondary years. Dynamic geometry linked to real world images

.or used to create dynamic simulations provided opportunities for students· to collect real or simulated data and to gain mathematical understanding through exploration using multiple representations.

Tasks which accessed these features proved to be valuable for both increasing students' engagement and their depth of mathematical thinking. The colour. movement and interaction created a halo effect valued by teachers for its impact on stu- dents' general attitude towards studying mathematics.

Teaching fourteen and fifteen year olds mathematics presents many challenges. Engaging these students can be difficult, especially as the mathematics they study becomes more abstract. A range of technology, now commonly available, offers opportunities to bring the real world into the mathematics classroom, to add visualisation, colour and animation not possible in a traditional classroom and to deepen the mathematical think- ing we expect in various topics of the curriculum.

The three examples of curriculum materials presented in th.is paper were developed as part of the RITEMATHS project (HREFI) and focus on some of the affordances of dynamic geometry packages. The project aimed to investigate the use of real (R) world conte)..'t problems with the assistance of Information Technology (IT) to enhance (E) middle secondary school students' engagement and achievement in mathematics (MATHS).

Increasingly mathematical software is becoming. integrated so that dynamic geometry may be seamlessly linked to a scientific calculator, lists or spreadsheet. a function grapher and perhaps a computer algebra system. Such packages may also allow for word processing and the use of digital images. These facilities allow the teacher to:

- give high quality, attractive presentations,
- bring real world situations into the mathematics classroom,
- set tasks that allow students to explore mathematical regularities and variation within one mathematical representation,·
- set tasks that allow students to explore mathematical ideas by linking different mathematical representations of, for example, the same function.

* University of Melbourne. Victoria 3010, Australia

The facility to quickly prepare high quality. attractive presentations is an advantage for the teacher. Such presentations not only attract students' attention but. when pre-prepared, may lessen the immediate cognitive load for the teacher as they deal with the class, the mathematics and the technology.

The value of real world problems has been well recognised (see for example Burkhardt, 1981). Linking to the real world has a cognitive impact as students learn that mathematics has both meaning and real world consequences. Such problems:

- foster general competencies and attitudes;
- prepare citizens who have critical competence;
- equip students with the skills to utilise mathematics for solving prob- lems;
- give students a rich and comprehensive picture of mathematics and
- can prompt mathematics learning of a traditional type {Blum and Niss, 1989).

In addition Pierce and Stacey (2006) report that teacher choices of real world problems enhance the image of mathematics by taking advantage of a 'halo effect'. The term 'halo effect' dates back to the l 920's work of Edward Thorndike (1920) showing that assessments of specific traits of a person or thing are markedly influenced by an overall impression, which may itself be based on little evidence. In marketing, the term 'halo effect' is used to describe the phenomenon of extending a positive view of one attribute or item to an entire brand.

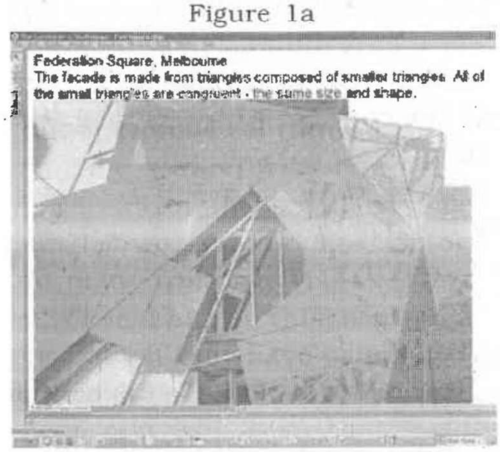
We have observed that many mathematics teachers' take advantage of this psychological strategy by associating mathematics with simple pleasures from real world contexts. They aim to capitalise on students' appreciation of colour, fun, and pleasant shared experiences to create a 'halo' effect that extends from the pleasant experience to the immediate mathematical task and beyond to the study of mathematics in general.

The real world may be brought info the mathematics classroom in two ways: through the use of digital images and through the use of simulations. Both of these can occur using dynamic geometry. In this paper the first two examples illustrate the use of digital images while the third illustrates the use of simulation. In. the sections below each example will be illustrated and discussed with attention to both the cognitive and affective aspects of student learning which are built into the design.

Example 1: Federation square and the golden ratio

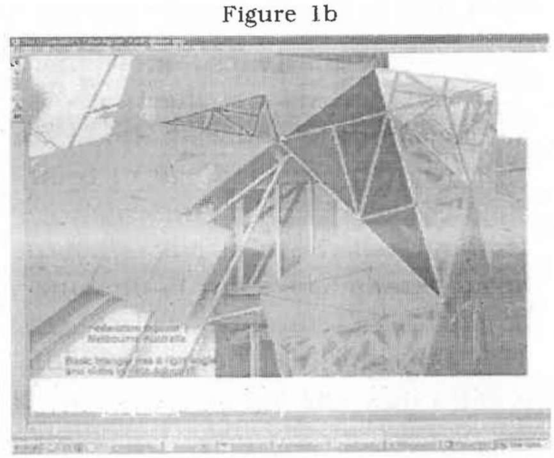
At Federation Square in the centre of the city of Melbourne there is a facade made of 22000 zinc, glass and sandstone triangles. Small triangles are progressively arranged. into larger geometrically similar triangles, using a famous pinwheel tiling. Figure 1 shows both a section of the facade and the same section with line segments used to highlight the pattern of tri- angles. All of the small triangles are congruent and all the highlighted triangles are similar. for Victorian students this is a familiar landmark that many will have visited. For many students it will evoke shared memories of looking at this feature as part of a school excursion to the city especially if they have completed the city 'maths trail' (Vincent, 2007). The mathematics is linked to the real world because the students have seen the facade and will learn that the fascinating and aesthetically pleasing at- tributes of the patterns in the facade are linked to the mathematical properties of the shapes used.

Figure 1a



THE FAÇADE OF FEDERATION SQUARE. MELBOURNE. USES A PINWHEEL TILING

Figure 1b



FAÇADE WITH THREE SIZES OF TRIANGLES HIGH-LIGHTED.

We have prepared a dynamic geometry file (in this case a Geometer's. Sketch Pad (GSP) file) that allows the user to access some of the functional opportunities afforded- by the software. It takes advantage of the facilities to insert pictures; create points, lines and polygons in front of the image; create triangles with specified mathematical properties, measure the length of lines; and perform calculations. Two sorts of constructions are used in the file: (i) triangles drawn on the photo highlight features as in Figure la (where right angles in the real world may not be right angles on the photo) and (ii) constructed right-angled triangles with side lengths that are multiples of 1, 2,v5 represent the triangles of the theoretical tiling (see Figure le). Investigation of the mathematical properties of the triangles in the facade prompts discussion of properties of triangles, measurement, congruent triangles, similar triangles, tessellations and ratio. If students know that the triangles highlighted in Figure 1b are all similar and right-angled, then it.is easy to see that the side lengths are in the ratio l:2:v5 (because the sides of a large ·triangle are made from 1 and 2 copies of the hypotenuse of the next smaller triangle).

None of these images are dynamic. What, then, does the use of dynamic geometry software in this way offer for teaching? It is true in this case that the construction, investigation and mathematical operations could be carried out by students using photocopies, pen, paper, ruler, scissors and calculator. The dynamic geometry _programs offer the opportunity to. project the photo of the real facade and examine it, with whole class discussion, within the orderly confines of the classroom. The material shown here was used in conjunction with paper measuring. which students could further link to the real world objects. Olivero and Robutti (2007) point out that discussion of measuring in _the real world and the digital world along with discussion of the theoretical mathematical world provides an opportunity to exploit what Pierce and Stacey call the contrast between ideal and machine mathematics. In ideal (theoretical) mathematics the length of the sides of the triangles will be exactly represented by multiples of 1, 2, and v5 . When the lengths are measured using the dynamic geometry software the results will be approximate and the accuracy reported according to the number of decimal places set.

In addition although the triangles are congruent, measures for equivalent sides may slightly vary with the orientation of the triangle (a software . phenomenon), the relative position of the camera and the real walls of Federation Square, and the camera lens properties. Some of this va1iation requires discussion with the students. Further, if students have hand drawn or cut out triangles, measures with their rulers will be limited by the accuracy of the ruler. If students are to use their mathematics to solve real problems then it is vital that they learn to understand the practical constraints associated with measurement, whether it is using real world objects such as a rule or on screen measurements such as through dynamic geometry.

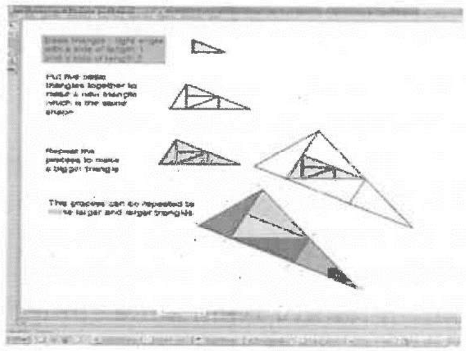


Figure 1c. Illustration of how to make the pinwheel tiling.

Dynamic geometry may be used to encourage students to develop a sense of discovery as active learners.· Azzarello et al {2002) describe the ascending and descending modalities of a lesson using dynamic geometry: ascending as the learning activity focus moves from perception of drawings to analysis through theory then descending when the focus moves from theory to drawings. This is reminiscent of the stages of mathematical modelling, as problems are transformed from problems in the real world, to problems in the mathematical world, solved and then having the solution interpreted and evaluated in real world terms. His also reminiscent of the horizontal and vertical mathematising of RME (realistic mathematics education). Descriptive names have been given to the various ways in which the user may use dragging to explore mathematical regularity and variation. Azzarello et al (2002) name 'wandering dragging', 'bounded dragging', 'guided dragging', 'dummy locus dragging', 'line dragging', 'linked dragging' and 'dragging test'.

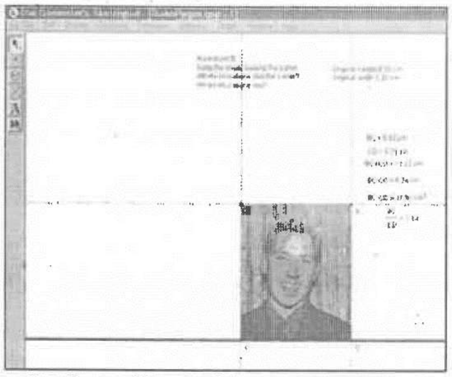
A number of these dragging modalities are illustrated by the examples which follow. It is important to note that from classroom research Azzarello et al note that the students· solution processes develop through a sequence of different modalities following an evolution from perceptive to theoretical through ascending and descending modalities. Exploratory tasks require careful didactical design to support students in such a learning process. Sinclair (2003) found value from using carefully prepared dynamic geometry files that encourage discovery without confusion and which can be delivered within a limited time frame. Such files should also be accompanied with carefully designed questions that help students "learn how to use change to explore and_ how to extend their visual interpretation skills" {p312).

Example 2: Digital images, aspect ratio and similarity

The world of digital cameras and resizing images is familiar to our students. So too is the variation in aspect ratio which must be considered When watching DVDs on computers, televisions or large screens. We have used these real world experiences tq introduce ideas :Of similarity using rectangles {rather than the classic approach with triangles) with "looks the same" as the fundamental meaning given to similarity. Digital images pasted into a dynamic geometry package, such as those shown in figure 2, offer links between mathematics and the real world. Students can manipulate the images, deciding whether they 'look the same' when the size changes. Through various modalities of dragging {as discussed above) students explore the regularity and variation of the length and breadth and also the _ ratios; sums and differences of these measures, while observing the changing image. When the image 'looks the same' the ratio of the length and breadth remains the same as the original. but the actual measurements and other quantities calculated from them alter. These experiences may impact on cognition in the areas of measurement, calculation, ratio, aspect ratio and similar figures. Positive affective impact may be gained by the choice of photos. These could be taken around the school or include images of the· students from the class. In this example different· dragging modalities are used to develop theory and test theory and these are structured into the design of the dynamic geometry worksheets.

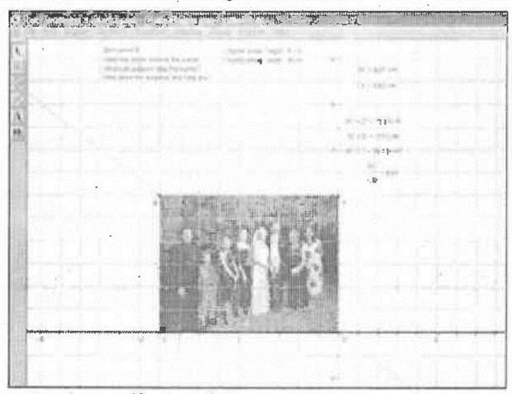
In the worksheet illustrated by figure 2a students are asked to drag the corner of the photo so that the image 'looks the same'. They are encouraged to use 'wandering dragging' and observe the effect on the shape of the image and on the lengths; ratios, sums_ and differences of the side lengths. From this experience they form conjectures about the preservation of aspect ratio. Initially, for example, students find that the images that look the same all have the same ratio of length and breadth. They also usually note that the one dragged corner stays on an (extended) diagonal of the image. Putting a trace on the dragged corner reveals this. This property can be linked then to the invariant ratios of the sides, and the equality of the factors the stretch or shrink the length and breadth. The dynamic geometry task has a series of worksheets. Figure 2b comes later in the series and provides both the grid and a diagonal line. This may either be used as a guide for students who have not developed a theory or the grid and line also make it easy for students to test their own theory for the preservation of aspect ratio. Other examples of the use of dynamic geometry and other image software are given by Pierce et al (2005).

Figure 2a



Explore effects of dragging corner of image-wandering dragging encouraged.

Figure 2b



Explore effects of dragging corner of image-bounded and guided dragging

Example 3: The biggest volume for a box made from A4 paper

The third and final example. requires pre-calculus students to explore an optimisation problem: what is the maximum Volume of an open box that can be made from a sheet of A4 sized paper or card? Our teachers have found that the impact of this task is greatest, and is remembered by students when they learn calculus some years later, if they each draw an appropriate net and construct a box and find its volume and plot a point on a scattergram of class results. This practical and numerical work pre- cedes the experiences with the dynamic geometry and also precedes the algebraic formulation. Greatest affective impact is gained by using brightly coloured paper, and stacking the boxes as in figure 3a thereby exhibiting their decreasing bases, and increasing height. The difficulty of judging which box has the greatest volume must be resolved using mathematics - the stacking does not help with that.

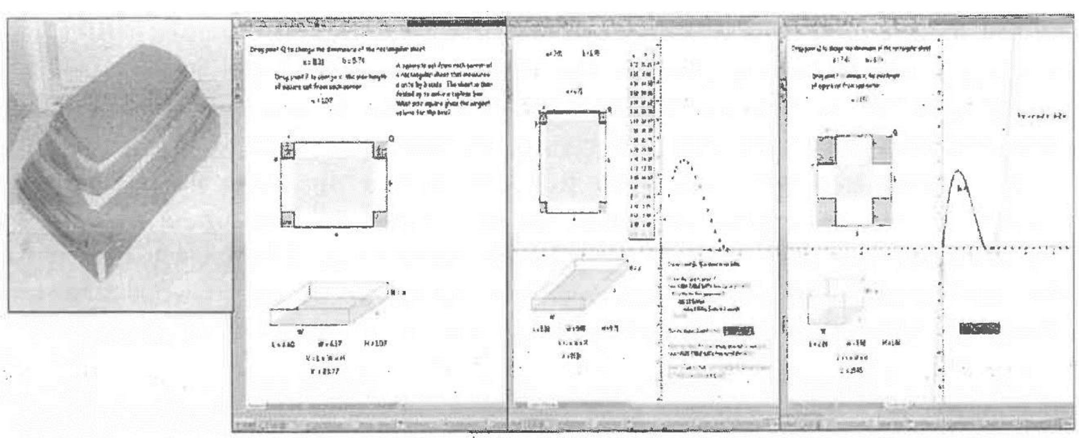


Figure 3a (far left) Coloured open boxes each-constructed from A4 paper. Figure 3b (mid-left) Dynamic file of open box and its net. Figure 3c (mid-right) Adding table of values and graph of height and volume. Figure 3d (far right) Open box, net and graph of volume function showing limited domain.

Because dynamic geometry is now linked with data capture, data graphing, list construction, its use enables students to:

- examine and explore a virtual model of the paper cutting, fully mathematised, which corresponds to their paper model but is manipulable in size;
- obtain first hand virtual experience of how the volume changes when the height of the box is altered (with the kinaesthetic of dragging);
- capture data to create a table of values of the height and volume (this step highlights empirical aspects of mathematical discovery);
- display the data graphically as isolated points, and as a curve:
- test whether formulas derived deductively match the data.

CONCLUSION

The examples above have illustrated three different types of uses of dynamic geometry. In the first, the dynamic geometry was especially used to enhance the presentation of the material. Students could have used the traditional tools of the pen-and-paper classroom, but the computer added positive associations, colour and clarity. In our research project, we particularly noted the very strong emphasis that teachers placed on these superficial' uses of technology, simply to brighten the classroom and improve attitudes to mathematics. Our aI1alysis of this phenomenon Jed us to ,the psychological literature on the halo effect (Pierce c1ud Stacey, 2006}. We now understand that an• important motivation for teachers to use· real world problems, which was previously neglected in the literature (see for example, Blum and Niss, 1989) is to associate mathematics with pleasurable parts of students' lives. Teachers hope (maybe not consciously) that using pleasant contexts, images and objects in mathematics lessons will, by association, make students feel more positively disposed towards learn- ing mathematics. These. presentation aspects of dynamic geometry and other software may (quite literally) be superficial, hut it does not mean they are not important.

The second example, introducing ideas of similarity through aspect ratio, inherently required technology to manipulate images. Azzarello's research group had identified the roles of different types of dragging in conducting investigations, and it is interesting that several of these. had been independently created for use in this unit.

In the third example, we see the expansion of multiple representations in algebra that is supported by dynamic geometry. Classic writing on multiple representations for algebra (see, for example, Kaput, 1992) has considered three representations (e.g. the numerical, symbolic and graphical representations of a quadratic function), but with dynamic geometry used to study real world problems we move seamlessly between five worlds:

- real world situation:
- dynamic geometry simulation;
- numerical representation;
- graphical representation;
- symbolic representation.

In conclusion, dynamic geometry as a stand-alone application, and even more as part of a package of mathematical tools has much to offer for teaching mathematics through real world problems.

Acknowledgement

The tasks described in this paper were developed as part of the RITEMATHS project (HREF1) The researchers thank the Australian Research Council, our six partner schools and Texas Instruments for their financial support of this project and especially the Teachers and students who took part in this study.

References

Arzarello. F.. Olivero, F., Paola. D., & Robutti, 0. (2002). A cognitive analysis of dragging practices in Cabri environments. Zentralblatt fur Didaktik der Mathematik. 34 (3) 66-72.

Blum, W., & Niss, M. (1989) Mathematical problem solving, modeling, appllcations; and links to other subjects - . State, trends• and issues in mathematics instn1ction. In \V. Blum, M. Niss and I. Huntley (Eds) Modelling, applications and applied problem solving. (pp 1 - 21). Chichester: Ellis Horwood.

Burkhardt, H. (1981} The real world and mathematics. Glasgow: Blackie HREF1 RITEMATHS project website. The cAcces0sed 1st June 2008 from http:// extranet.edfac.unimelb.edu.au/DSME/RITEMATHS/

Kaput, J. J. (1992}. Technology and mathematics education. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 515- 556). New York: Macmillan.

Olivero, F. & Robutti, 0. (2007). Measuring in dynamic geometry environments as atool for conjecturing and proving. International Journal of Computers for Mathematical Learning. 12 135-156.

Pierce, R & Stacey, K. (2006). Enhancing the image of mathematics .by association with simple pleasures from real world contexts. Zentral blattfilr Didaktik der Mathema,tik. 38(2) 214-225.

Pierce, R., Stacey, K., & Ball, L. (2005) Mathematics from still and moving images. Australian Mathematics Teacher. 61(3), 26 - 31.

Sinclair, M. (2003). Some implications of the results of a case study for the design of pre-constructed, dynamic geometry sketches and accompanying materials. Educational Studies in Mathematics. 52, 289-317.

Thorndike, Edward L. (1920) A constant error in psychological ratings. Journal of Applied Psychology.• 4. 25 - 29.

Vincent, J. (2007). Shrine to University: A geometry journey along St. Kilda Road and Swanston Street. Melbourne: Mathematical Association of Victoria.