Research Article

AN OBJECTIVE MEASUREMENT OF SELF-MONITORING STRATEGY ON PROBLEM SOLVING IN MATHEMATICS

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Abstract

Approaching Mathematics through problem solving can create .a context, which simulates real life and therefore justifies Mathematics rather than treating it as an end in itself. Problem-solvers intend to employ a combination of domain-specific knowledge and strategic knowledge. Researches indicate that students problem solving failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know. This paper describes a study that investigated the self-monitoring strategies used by high school students while working individually on a Math problem. Identifying the characteristic type of metacognitive failures of solution highlighted the distinction between the two key elements of effective monitoring: being able to recognize errors and other obstacles to progress, and being able to correct or overcome them.

Keywords: *Problem Solving in Mathematics, Real-Life Context, Domain-Specific Knowledge, Strategic Knowledge, Self-Monitoring Strategies, Metacognitive Failures, and Error Recognition and Correction.*

Introduction

In review of progress in problem solving research over the past 25 years, Lester (1994) noted with some concern that research interest in this area appears to be on the decline, even though there remain many unresolved issues that deserve continued attention. One such issue highlighted by Lester was the role of self-monitoring in problem solving- where self-monitoring refers to what students know about their own thought processes, and how they regulate their thinking while working on problem solving.

Although the importance of self -monitoring is now widely acknowledged, teachers in India still lack an adequate theoretical model for explaining the mechanisms of individual self-monitoring and self-- regulation.

The study reported here examined the individual problem solving actions of a group of high school students. The results reported here provide glimpses into the mathematical thinking of individual students as well as suggesting follow up investigation into the nature of collaborative monitoring and regulation.

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Research Questions

The specific research questions addressed in this study are as follows:

1. What strategies do students use in attempting to solve a problem?
2. How do students recognize and respond to obstacles to progress?
3. How is metacognitive self- monitoring related to problem solving outcomes?

Method

Subjects

Forty-two students of standard IX from Christian Girls Higher Secondary School, Tanjore were selected as sample for the present study. Tl;:l.e decision to focus on high school mathematics class rooms was prompted in part by the introduction of revised·syllabus. The traditional emphasis on memorization and basic skills has given way to arguments that students also need to develop reasoning and problem solving capacities. The incorporation of these goals into high school teachers to re-examine their conceptions of mathematics learning and teaching; hence one of the aims of the research study was to explore the implications of the problem solving for classroom practice.

Questionnaire

The self - monitoring Questionnaire elicited students' retrospective reports on the metacognitive strategies they had employed while working on a given mathematics problem. The Questionnaire is based on an instrument used with B.Ed. trainees having choices Yes/ No by Ramganesh (2003). To make the questionnaire more appropriate for standard IX students, the original version was modified by deleting, rewording. and including some items with the consultation of experts in the field of Education and Cognitive Psychology. In the present study, the questionnaire consisted of fifteen statements to which students responded by ticking boxes marked Yes, No or Unsure

The first response sheet of the questionnaire, titled "Before you started to solve problem", listed six possible strategies concerning reading and understanding the problem. The second response sheet, "As you worked on the. method", listed five possible strategies concerning analysis and execution of a solution; while the third, "After you finished working on the problem" offered four strategies for verifying the solution.

The Self-Monitoring Questionnaire implicitly investigate .students' ability to

•• recognize and act on metacognitive warning signals. arising during routine monitoring, which indicate the need for regulation or repair. Each questionnaire statement is identified as a generic type of metacognitive self-monitoring or self-regulatory activity, as used in the framework of Figure

1. In addition, questionnaire statements that target the "red flags" of error detection, lack of progress, and anomalous result are identified.

Self - Monitoring Questionnaire Item	Monitoring / Regulation
Before your started 1. I read the problem more than once. 2. I made sure that I understood what the problem was asking me. 3. I tried to put the problem into my own Words. 4. I tried to remember whether I had worked on a problem like this before. 5. I identified the information that was given in the problem 6. I thought about different approaches I could try for solving the problem	Assess knowledge Assess understanding Assess understanding Assess knowledge & understanding Assess knowledge & understanding Assess strategy appropriateness
As you worked 7. I checked my work step by step as I went through the problem 8. I made a mistake and had to redo some working. 9. I re-read the problem to check that I was still on track. 10. I asked myself whether I was getting any closer to a solution. 11. I had to rethink my solution method and try a different approach.	*"Red Flag": Error detection* Assess strategy execution Correct error "Red Flag": Lack of progress Assess understanding Assess progress Assess strategy appropriateness change strategy

After you finished	*"Red Flag": Anomalous result*
12. I checked my calculations to make sure they were correct.	Assess result for accuracy
13. I looked back over my solution method to check that I had done what the problem asked.	Assess strategy appropriateness and execution
14. I asked myself whether my answer made sense.	Assess result for sense
15. I thought about different ways 1 could have solved the problem	Assess strategy appropriateness

Figure 1. Metacognitive strategies examined by Self-Monitoring Questionnaire.q3b

Task

Because the aim of the questionnaire was to gather data on self monitoring strategies rather than simply assess mathematical expertise, it was important to supply a genuine "problem" that would challenge the students and call forth processes of interest, without requiring any specialized mathematical knowledge'. These criteria were proved . to be satisfied by the problem on simultaneous equations as follows:

Solve

$$3x + y + z = 3 \text{ ——— } 1$$
$$2x + 2y + 5z = -1 \text{ ——— } 2$$
$$x - 3y - 4z = 2 \text{ ——— } 3$$

Students are made to recall their previous knowledge about solving linear equations having one variable and two variables. In general, they are expected to understand that anything which is unknown (solving simultaneous equations having three variables) can be found out by recalling the known rules {solving linear equations with two variables or one variable)(Advanced organizer).

Now students are expected to acquire concretized knowledge .and initial prediction about the problem. They are also confident of proceeding the problem.

Teachers presents the diagrammatic scheme of the problem as follows. (Problem solving strategy)

Strategy #1: ***Random selection :***

Solving (1) & (2)

or

(1) & (3) Reduce a -------> take ------> (4)

or variable

(2) & (3)

Strategy #2: ***Formulation of Reducing a variable***

Solve two of the equations not taken already

Reduce a variable which is already-----> take-----> (5)

reduced in rule (1)

Strategy #3: ***Finding the value of a variable***

Solve (4) & (5) -----> Reduce a variable -----> Find the value of a variable

Strategy #4: ***Substitution***

Substitute the values of two variables in either (1) (2) or (3) whichever have small coefficients -----> Find the value of the third variable

Strategy #5: ***Logical Reasoning***

List the value of all the variables

To bring awareness of one's attention the following questions are asked;

i. What do you understand to solve these simultaneous equations?
ii. Do you understand how to proceed?

They are motivated to respond and they attend the stimuli of the task

Students are instructed to use the same strategy they use for solving linear equations having two variables, about doing Strategy #1. So students are guided to choose the appropriate strategy which they know already.

(1) x 2 => $6x + 2y + 2z = 6$

(2) => $2x + 2y + 5z = -1$

$4x - 3z = 7$………….(4)

To bring one's awareness and self-control of his understanding which variable is to be reduced to get equation (5) by taking (2) and (3) or (1) x (3) (probing) Students are expected to develop logical reasoning and they respond the question.

(l)X 3 9x+3y+3z =9

X -3y-4z =2

10x -z =11…………(5)

Now, students are given verbal direction to thinking (problem solving strategy)to proceed next;

In Strategy #3. Students are to solve for equation (4) and (5). Also they are given orientation to select the ongoing experience (orchestration) to do the task Strategy #3, as to reduce another variable.

(4) => 4x - 3z = 7

(2) => 30x - 3z = 33

-26x = -26

x=l

Now

Substitute x=1 in (4)

4-3z = 7

-3z = 3

z = -1

Students are helped to regulate their thinking (self-regulation) about how the variables have been reduced as .variables into two and two variables into one.

Now they learn to determine the order of steps to be taken to complete the task (organizing) and also the speed at which they should work this type of problem (self- regulation).

It was anticipated that students would attempt a combined algebraic/ trial and error solution. A skilled formal approach would resemble that shown above.

Students were given the written problem statement and allowed twenty to thirty minutes for working. They · were instructed to show all their working and to cross out, rather than erase, any working which was incorrect. • Only at the end of this time was the questionnaire administered, to avoid cueing students on the strategies it listed.

Questionnaire Responses

A high rate of Yes responses was recorded for almost all Self- Monitoring Questionnaire statements referring to metacognitive strategies. Response rates for the four statements that might prompt initial recognition o(the metacognitive "warning" described earlier (lack of progress, error detection, anomalous result - see Figure 1) are shown in Table 1. While these results seem to suggest that students were immersed in metacognitive activity. it is unwise to accept self-reports of this kind at face value as information relating to regulation of cognition is not necessarily stable (Brown, Bransford, Ferrara & Campione. 1983). The students' questionnaire responses therefore must be interpreted in the light of their actual problem solving behaviour.

In the next section students' written solution attempts are examined and, where necessary, compared with their responses to the questionnaire statements in Table 1 to reveal self monitoring successes and failures.

Table 1: Questionnaire Responses to Metacognitive "Red Flag" Statements

"Red Flag"	Questionnaire Statement	Percentage of Students Responding *Yes*
Lack of progress	I asked myself whether I was getting any closer to a solution.	81%
Error detection	I checked my work step by step as I went through the problem	63%
Anomalous result	I checked my calculations to make sure they were correct	77%
	I asked myself whether my answer made sense.	84%

Successful Self-Monitoring

Successful self-monitoring is difficult to detect if it merely confirms that satisfactory progress is being made. However, the students' written work did provide evidence of self-monitoring where difficulties or errors forced a change of strategy. For example, although Table l shows that eight of the fifteen students who used a mean value strategy nianaged to find one of the answers to the problem, it does not reveal that six of these students began working with a different strategy that was subsequently abandoned. In three of these cases, the change of strategy was caused by lack of progress in formulating the problem algebraically. The remaining students discarded their initial strategy because it produced an answer that was either unreasonable or inaccurate. There was also some evidence that other students rejected unreasonable answers. but were unable to identify an alternative strategy.

Failures in Self-Monitoring

Examination of Table 1 shows that there were three broad groupings of solution strategies and outcomes:

1. Inappropriate strategies that gave incorrect answers.
2. Inefficient strategies through which it was possible, with luck and persistence, to find one answer, but that were equally likely to result in no answer being found at all.
3. Appropriate strategies that had the potential to produce one or both answers, provided that the strategies were correctly executed and a way was found to solve equations with two unknowns.

Analysis of individual students' solution scripts and questionnaire responses showed that the above strategy and outcome groupings were associated with corresponding failures to recognize, or act on, the metacognitive "red flags" described earlier:

1. Anomalous results were verified and accepted
2. Lack of progress towards obtaining an answer did not lead to a change of strategy.
3. Errors in strategy execution remained undetected.

Each of these failures in self-monitoring is described in more detail Table 2.

Table 2 : Evidence of Self-Monitoring in Users of Inappropriate Strategies· (Incorrect Answer)

Evidence from Written Work	Evidence from Questionnaire						
	Checked Calculations			Checked Sense			
	Yes	No	Unsure	Yes	No	Unsure	Total
No evidence of verification	2	1	0	3	0	0	3
Faulty verification procedure	1	0	0	1	0	0	1

Verified non-integral	4	0	0	2	0	2	4
Verified integral	4	1	1	4	1	1	6
Total	11	2	1	10	1	3	14

Note : Sixteen of the total of 42 students were categorized as using inappropriate strategies: Of these, fourteen obtained an incorrect answer. (A further two students obtained no answer).

Of the sixteen students who used an incorrect formulation or assumed, fourteen obtained incorrect answers, that is, an answer that violated the problem conditions. Since an incorrect answer represents a metacognitive warning signal that should trigger a review of both the accuracy of calculations and the appropriateness of the str:c1tegy. ."t is tempting tv assume that these students did not try to verify their answer. However.evidence from their questionnaire responses and written work, summarized in Table 2, suggests otherwise; Eleven students claimed that they checked their calculations, and ten reported that they asked themselves whether their answer made sense (Table 2-Evidence from Questionnaire). In most cases, their written work confirmed that they did indeed carry out some kind of verification procedure (Table 2-Evidence from Written Work); however, many appeared to accept either an integral answer that did not satisfy the problem's explicit conditions, or a non- integral answer that did not make sense.

Table 3 : Evidence of Self - Monitoring in Appropriate Strategy Users (Incorrect Answer)

Evidence from Questionnaire										
	Checked working			Checked Calculations			Checked Sense			
Evidence from Written Work	**Yes**	**No**	**Unsure**	**Yes**	**No**	**Unsure**	**Yes**	**No**	**Unsure**	**Total**
Incorrect answer caused by undetected errors	1	1	0	2	0	0	2	0	0	2

Note : Nine of the total of 42 students were categorized as using appropriate strategies.
Of these, two obtained an incorrect answer. (A further two obtained no answer, and five obtained one or both answers).

Nine students used algebraic or verbal reasoning strategies. Five were at least partly successful, obtaining one or both answers. and another might have found an answer if she had taken more time in systematically trialing x and y values. The other student who failed to obtain an answer was hindered by her persistent, _and fruitless, attempts to eliminate one of the two variables from the equation she had derived. Interestingly. this student stated that she was "unsure" whether she had assessed her progress towards a solution (questionnaire response). Despite using an appropriate strategy, a further two students obtained incorrect answers. Evidence from their written work and questionnaire responses is summarized in Table 3. Both these students recognized treat their answers were incorrect and / or unreasonable, but they failed to detect simple algebraic errors either while they were working on the problem or later when they checked their calculations.

Conclusion and Implications

The aim of the study reported here was to investigate the metacognitive self- monitoring strategies used by secondary school students while working individually on a mathematic "problem" (i.e. a task that presented obstacles to their progress). Students' self- monitoring activity was inferred from their written work on the problem and questionnaire responses.

Examination of the questionnaire responses and written working of the students whoattempted the problem revealed connections between solution strategies, outcomes and self-monitoring. Analysis centred on identifying students' recognition of three metacognitive warning signals: lack of progress, error detection, and anomalous result.· Ideally, each should prompt a reassessment of either the appropriateness of the chosen strategy, or the manner in which it was executed. Thus, expected recognition and response patterns are as follows:

1. Students using inappropriate strategies •that lead• to incorrect or unreasonable answers should check their calculations for errors and. if none are found, consider a change of strategy;
2. Students using appropriate strategies that, nevertheless. produce an incorrect answer should find and correct their errors.

In practice, only five students used appropriate strategies (algebraic or verbal reasoning, supplemented by principled testing · of values for one variable) leading to one or both answers being obtained.

Although there were instance of successful self - monitoring, it was found that many students were either oblivious to the warning signals mentioned above, or were unable to act appropriately if the signals were detected. Even if students do review their progress towards the goal, check their calculations while they work,· and attempt to verify the accuracy and sense of their answer, their worthy metacognitive intentions will be foiled if they are unable to recognize when they are stuck. have no alternative strategy available, cannot find their error (or cannot fix it if they do find it). or fail to recognize nonsensical answers;

The problem of recognizing difficulties is clearly illustrated in the work of students who "verified" answers that either contradicted the information given in the problem or made no sense in real world terms. Although it is possible that these students had misgivings that they did not record, one wonders whether their years of schooling have engendered a belief that school mathematics tasks need not make sense. Ironically, some of the students who did explicitly reject these kinds of answers could not think of any other way to attack the problem. If teachers wish to encourage students to monitor and regulate their mathematical thinking, it is important to ensure not only that they are attuned to the signals that alert them. to danger, but also that they are well equipped to :respond.

In interpreting these results. teachers should not lose sight of the fact that the problem was chosen for use in this study because of its challenging nature - that is, it was hoped that the task would raise the types of obstacles referred to above so that metacognitive strategies would be called into play. Perhaps, then, it is not surprising that so few students succeeded in obtaining a complete solution, or in effective monitoring and regulation their problem solving activity. If fact, we have observed similar results with pre- service teacher education students and practicing teachers who have tackled this task in professional development workshops. Teachers deserve many such opportunities to analyse their own mathematical thinking and consider implications for classroom practice if they are to successfully implement current curriculum policies promoting reform in mathematics education.

From·. a methodological perspective, it is acknowledged that questionnaires should be used with due control in isolation to investigate metacognitive strategy use. For example, the findings reported here deserve follow up via individual student interviews that probe questionnaire responses and seek explanation of the solution strategies they adopted. Further research is also needed to investigate the strategies students apply in regular classroom settings, when peers become an additional resource for tackling obstacles to problem solving progress. Such a research activity should also consider implications for teaching in particular, how mathematics teachers can develop metacognitive abilities in their students. The Self-Monitoring Questionnaire, when used in conjunction with a suitably challenging task, is a pedagogical tool that teachers could use with their own classes to extend students' repertoire of metacognitive strategies, and to understand the revolution of problem action.

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