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# INDIAN EDUCATIONAL RESEARCHER

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# CONTENTS

# EDITORIAL

Indian Educational Researcher has now entered its third year in existence. The changes the journal has been through since its advent has been a tremendous learning experience for all of us at Stella Matutina College of Education.

Volume 3 Issue 1 represents the true spirit of the journal both in terms of its content and the geographic reach. I am truly proud to present to the readers this issue focusing on Mathematics Teacher Education with contributions from internationally reputed Mathematics Teacher Educators. Dr. Jill Adler addresses the status of Mathematics teacher education in South Africa. From the University of Melbourne, we have Dr. Robyn Pierce and Dr. Kaye Stacey discussing the place of dynamic geometry in the middle school mathematics curriculum. Dr. Shweta Naik from the Homi Bhabha Centre for Science Education has examined the understanding of non-typical examples in mathematics teachers. Dr. Ramganesh from Bharathidasan University has investigated self-monitoring strategies used by high school students while problem solving. Ms. Kanmani and Dr. Annaraja report on the relationship between metacognition and academic achievement of computer science students. Musings has Mr. Sundaram, Principal, KD Ambani Vidyamandir, and an experienced mathematics teacher, pondering on why students find mathematics dif1lcult.

Thank you for your support. We look forward to continued active interaction from our readers.

**Radha Mohan**

Associate Editor

Conceptual Article

# Mathematics teacher education in South Africa: A research agenda focused on the mathematical work of teaching across diverse contexts

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**Abstract**

Mathematics teacher education in South Africa faces unique challenges shaped by the country's social, cultural, linguistic, and economic diversity. Preparing teachers who can effectively engage with learners across such varied contexts requires a research agenda that foregrounds the mathematical work of teaching (MWT)—the specialized content knowledge, pedagogical reasoning, and classroom practices that enable effective mathematics instruction. This paper outlines a research agenda that emphasizes three key areas: (1) understanding how teachers engage with mathematical concepts and learners' reasoning in multilingual and resource-constrained environments, (2) exploring the relationship between teachers' professional knowledge and their instructional practices, and (3) examining the institutional and policy frameworks that shape teacher preparation and continuing professional development. The agenda calls for empirical studies, context-sensitive methodologies, and collaborative approaches that integrate teacher educators, researchers, and practitioners. By situating the mathematical work of teaching within South Africa's diverse educational contexts, this research agenda seeks to inform policies and practices that strengthen teacher education, improve mathematics learning outcomes, and contribute to equity and quality in education.

**Keywords:** *Mathematics teacher education; South Africa; mathematical work of teaching; pedagogy; teacher professional development; educational diversity; multilingual classrooms; equity in education.*

## INTRODUCTION

This paper reflects on aspects of mathematics .teacher education in South Africa, with a focus on "the ·mathematical work of teaching". This brings the following questions to our minds.

1. What do mathematics teachers need to know to be able to do, to teach mathematics well?
2. How do the mathematical work teachers do. Differ across diverse contexts of teaching?
3. Can we arrive at a research agenda in support of all this?

The underlying theoretical assumption behind such a research agenda is that mathematical work is situated. This assumption is borne out by empirical studies of mathematics used in various workplaces where there is a specificity to how mathematics. is attuned to the needs and demands of varying cultural practices (Noss, 2002; Noss, Hoyles & Pozzi, 2001). Similarly, it is arguable that there is specificity to the mathematical demands of teaching. The difference, of course, is that teachers are trying to teach mathematics. This makes the mathematical demands of their work different from nurses, say, who use mathematics in the course of their nursing. Their work is to nurse others to health, and so not mathematical in its intentions and outcomes. This difference aside, there is growing support for the notion that there is specificity to the way teachers need to hold and use mathematics in order ·to teach mathematics - and that this way of knowing and using mathematics differs .from the way mathematician hold and use mathematics (Ball & Bass, 2000). As this knowledge has a practice base, more focused research on mathematics teaching is required. Otherwise it will remain under-described and not well understood. This has significant implications for mathematics teacher education: it raises questions on the mathematical education of teachers, whether it gears itself to these ways of knowing and using mathematics.

The resonance of a notion of the mathematical work of teaching being situated has practical and local roots in my work over many years in diverse classrooms in South Africa. There are diverse demands on teachers and teaching across ranging contexts, particularly those constituted by deep inequality and, as in now increasingly common elsewhere, by multilingualism. I have argued elsewhere (Adler, 2001) that there are context- related dilemmas of teaching in multilingual mathematics classrooms. This earlier research reveals that all teaching is dilemma-filled. However, South African mathematics teachers manage particular language-related dilemmas that are at once a function of both their personal biography and the varying contexts of their work. What that research did not explicitly foreground, however, was the mathematical work teachers did to manage these dilemmas. As is discussed later in this paper, secondary analysis of this data could reveal whether and how the mathematical work of teaching across contexts is indeed a fruitful field of research.

Issues of access and equity in education are both global and local. More and more, countries across the world are dealing with the economic and electronic divide, either Jqcally qr globally or both. And in many developed contexts, urban·schooling has come to be defined by economic and linguistic inequality. It follows that one methodological implication of a situated notion of mathematical work of teaching is that this critical field needs to be informed by research that is carried out in diverse classrooms. Simply put - the empirical sites for understanding the mathematical work of teaching matter.

This paper describes the circumstances (research-based and practical) that have led to this focal point in my research. The conditions in mathematics teaching and teacher education in South Africa described in this paper leads to the questions: Is. what teachers need to know and be able to do mathematically to teach in such conditions obvious? Is this knowledge in and for the practice of teaching well described and understood? This paper also looks at research that has been done, as this provides some of the rationale for the knowledge-for-teaching agenda, and also the basis on which such research can build. Some of this research is discussed later in this paper, with focus on the mathematical work teachers appear to be doing.

## CONDITIONS ON THE GROUND IN SOUTH AFRICA

Three questions frame the discussion of conditions on the ground in South Africa. Who are going to be, and who currently are, mathematics teachers in South Africa? For the purposes of this paper, and for more focused research these questions will have a Senior Phase (Grades 7 - 9) teacher in mind. What mathematics is selected into the Senior Phase curriculum i.e. what are teachers required to teach? Where are teachers going to teach?

**Who are / going to be mathematics teachers in SA?**

Currently, few graduates in mathematics are choosing to enter teaching in South Africa. Numbers in our Post Graduate Certificate in Education (PGCE). the usual route for secondary teacher qualification, have diminished dramatically in the past ten years. Shortages of suitably qualified secondary mathematics teachers in South Africa have reached critical propo1iions. Some might 'blame' this situation on the turmoil in post-apartheid education, and demoralization across the profession. However. this phenomenon is not peculiar to South Africa.

A four year undergraduate teacher education B Ed degree is now in place in the country, and, in my own institution, we have, relatively speaking, a reasonable intake of students. To date, three cohorts have graduated and these new teachers are qualified to teach mathematics across .secondary grades. The issue we face and deal with in the conceptualization and teaching of this undergraduate program is that these students, typically, did not perform particularly well in mathematics in .school. If they had, it is more likely they would have entered the Science Faculty. If they are to emerge from their studies with strong mathematical identities, these need to be produced and nurtured through their mathematics courses.

At the same time, the majority of black secondary teachers trained under apartheid only had access to a three year College of Education diploma. It is beyond the scope of this paper to explain just how poor, much of the quality of this training was ( welsch, 2002). Hence many secondary mathematics teachers currently in service have not had adequate opportunity to learn further mathematics. Here too, mathematics teacher education faces the challenge of working on intervention programs where in-service teachers can develop their mathematical knowledge and mathematical identities. What mathematics should be in such programmes? Where and how should these be taught?

The critical point here is that in both pre- and in-service mathematics teacher education programs, mathematical know-how and dispositions need to be produced, and in ways that will enable teachers to. project strong mathematical identities in their teaching. This is a considerable challenge, and contrary to assumptions that underpin secondary mathematics teacher education: that prospective secondary teachers already have a mathematical disposition and considerable mathematical competence that now needs to be tuned to the needs of teaching.

**What mathematics are Senior Phase teachers required to teach?**

Elsewhere, and drawing on Hargreaves (2001) and Elliott (2001), I have described the paradoxes facing teaching and teacher education in general (Adler, 2002). Teachers are being expected to meet multiple and competing needs simultaneously: for excellence and equity, for alleviating social ills, and performing well in competitive assessments. The list goes on. Graven (2002) brings some of these paradoxes to light for mathematics. In a detailed analysis of new Norms and Standards for Teacher· Education in South Africa, and the Revised National Curriculum Statement for Mathematics, Graven shows the multiple and competing roles and identities implicit for mathematics teaching in South Africa. Senior phase mathematics teachers are expected to induct learners into mathematical thinking (an investigative and problem-solving approach to mathematics is advocated through much of the policy documentation). At the same time, teachers are to appreciate how mathematics is and. can be used in real-life problem solving (that is, an . applied orientation to mathematics is simultaneously advocated). Moreover, given the history of South Africa, there is now an explicit demand for all education to tackle issues of democracy and human rights, and so built into the mathematics curriculum is the expectation that teachers will induct learners into a critical approach to the uses and abuses of mathematics, and to the skills needed for critical and participatory citizenship. All of these are to be bolstered by levels of procedural and computational fluency. These are what ·we could describe as a wide range of mathematical practices, and they are embedded in new and old topics in the curriculum (e.g. data handling - statistics and probability; transformation geometry are new curriculum topics). Neither these topics, nor an explicit focus• on mathematical practices are the typical fare· of mathematics courses in teacher preparation and development programs.

**Where are mathematics teachers teaching?**

Much has been written about inequality of education that was produced by apartheid education. We have, elsewhere, called the English Language Infrastructure in a school (Adler, 2001; Setati, Adler, Reed & Bapoo, 2002). Language-in- education policy advocates additive multilingualism. Yet, English remains the language of power, and so the preferred language of learning and teaching (LoLT) (Adler, 2001; Setati, 2002). Urban and non-urban schools in South Africa differ substantively in the extent to which English language is heard and used in and around the school.. In many non-urban areas, 'poverty prevails. There are few resources, including written texts. In addition, the dominant language in the region is the one in use outside of schooling. In all schools, teachers and learners aspire to English language fluency and so mathematics comes to be taught in and through English. The demands on teachers in non-urban contexts, are enormous as they are teaching in what can be described as Foreign Language Learning Environments (Setati .et al, 2002).

What is the mathematical work of teaching across such contexts? What are the linguistic and mathematical demands on teaching when the LoLT at the same time, is an object of study? What do mathematics teachers need to know and be able to do mathematically to teach this curriculum in these conditions? Do we know how to answer these questions effectively? What have we learned from previous research?

## WHAT HAVE WE LEARNED FROM PREVIOUS RESEARCH?

In the concluding chapter on a report of research on teachers' take-up from a formalized in-service program, Adler, Slonimsky & Reed (2002) posit that a central task for teacher research and development in South Africa is to better grasp what we coined "conceptual-knowledge-for-teaching." We arrived at this through a three-year, in-depth study of mathematics, science and English language teachers who participated in a formalized HY-service teacher development programme. Both in the programme and in the research, what remained elusive yet central to all explanations of take-up, be it in relation to use of resources, mediation or reflection on practice, was the ways in which teachers' struggled to elaborate the subject purposes of their work. We argued that a simplistic and typical response to this (e.g. Taylor & Vinjevold, 1999) is. that teachers do not know their subject (mathematics) well enough, and therefore need to do more· courses in mathematics. The simplicity of this interpretation leads to the kinds of formalised in-service programmes we have seen mushroom across universities in South Africa in the past few years. Teachers who were subjected to the inequities of apartheid teacher education (Welsch, 2002) are now provided upgrading possibilities - where they are being offered opportunities to learn 'more' maths, and .perhaps some mathematics education.

This response, however well-intended, fails to grasp the specificity and complexity of subject knowledge for teaching, and reinforces the suggestion made earlier, that this kind of knowledge is under-described. How does knowledge of andabout mathematics, for example, come to be used effectively in teaching? The problem here is not simply one of different pieces or kinds of mathematics teachers need to know, but critically one. of how it needs to be used to enable others to come to know mathematics. In the mid-1980s, and what can be described as a critical moment in the educational field of knowledge about the practice of teaching, Lee Shulman posited the notion of Pedagogic Content Knowledge (PCK) (Shulman, 1986, 1987). He pointed to the complex nature of knowledge-in-use in and for teaching, and the centrality of the integration of disciplinary or subject knowledge with knowledge. about teaching and learning for successful teaching. Ball & Bass (op cit) have done a great deal to• elaborate the nature of this mathematical work through their in depth study of an elementary teacher's work in a Grade 3 classroom over a full school year, and more recently through studies across a range of elementary classrooms in the United States (Ball et al, 2008; Hill et al. 2008). Like them, I posit that we do not know enough about this mathematical work that teachers do, and further that as this is practice_-based knowledge, we are faced with an empirical question. Just as Noss & Hoyles studied nurses' uses of mathematics in their day to day nursing work, so we need to study more systematically the what and how of mathematics in use in teaching.

As mentioned, Ball and her research colleagues have already contributed to such a research agenda. Indeed there are others researching subject knowledge for teaching with a specific focus on mathematics (e.g. Even, 1990; Kennedy, 1997; Ma, 1999). and these include studies at the secondary level. My own work in South Africa in the past few years has focused on mathematics for teaching, with particular interest in what is being offered in. teacher education {Adler & Davis, 2007; Davis, Adler and Parker, 2007; Adler & Huillet, 2008). Work in this field is thus already underway. What is its value in the South African context? Is there more that could or needs to be learned?

**Research on teaching and learning mathematics in multilingual classrooms**

Research on teachers' knowledge of the practices in urban secondary multilingual mathematics classrooms in South Africa (Adler, 2001) posits three dilemmas of teaching that describe such knowledge: dilemmas of code-switching (of enabling meaning through use of learners main language vs enabling access to English as the language of instruction, and the language of advancement); dilemmas of mediation (of valuing diverse learner productions vs production mathematical communicative competence); and dilemmas of transparency (of managing implicit and explicit mathematical language practices). The overarching argument is that. firstly, these dilemmas are at once personal and contextual, a function of the teacher's biography and the context in which thej' teach: and secondly that teachers manage these dilemmas in their day-to-day practice, making professional judgements as to how they work with many languages and diverse linguistic competencies present in their classrooms; Standing back from these dilemmas and the earlier, a question I now ask, (this was not in focus during that study) is: What mathematical moves do teachers make, or need to know how to make. in these moments? Is the mathematics teachers come to use attuned in any particular way to these diverse contextual conditions?

Setatl & Adler (2001) and Setati et al (2002) have described the complex journey that is or needs to be traveled in mathematics classrooms, from informal talk in learners' main language to mathematical writing in English, and the challenges for teachers in navigating this terrain. These challenges are at once a function of context, and more pertinently here, a function of working across multiple languages and discourses in the mathematics classroom. In the latter study of teachers· take-up from a formal in-service programme (Setati et al, 2002) we have shown how the English language infrastructure across urban and rural schools matters, and how teachers' navigation of the journey was largely incomplete or abbreviated. Their take- up from the programme in relation to language as a resource for learning and teaching resulted in a dominant practice of learners being afforded opportunity to use their main languages as a social thinking tool, and in informal talk as they began work on a mathematical task. The moves from there to mathematical talk and writing in English and mathematical English more particularly. were either absent (learner activity remained at that level} or abbreviated {with a radical jump by the teacher to .formalized mathematics in English). In the study we illustrated further that language• practices, particularly code-switching, differ across levels {elementary and secondary classrooms); across school subjects (teaching and learning English a language vs teaching and learning Mathematics of Science). The range of dimensions of diversity across contexts matters in the teaching and learning of mathematics.

And so here too the question arises as to whether there ·are specific mathematical entailments of teaching across diverse linguistic contexts. My work to date has not had this as an explicit focus. (How) do diverse contexts matter for the mathematical work that teachers do and need to do in order to teach mathematics well? Do they need to think about and work with mathematics in any specific ways so that they can enable their diverse learners, in diverse and often difficult conditions to learn? In simple terms, research· in teacher education, and on the. teaching and learning of mathematics in and across diverse (including multilingual) classrooms adds weight to the potential significance of the question of the mathematical work of teaching - context does matter. and it is and open and empirical question as to how this functions to· produce particular mathematical demands on teachers. It is thus important that the current work on subject knowledge for teaching and pedagogic content knowledge is carried out in diverse classroom contexts. and ther1 too with appropriate theoretical tools that will enable a gaze that holds the context and mathematics in their inter-relation in focus.

## A RESEARCH AGENDA

In this paper I have built an argument for specificity to the mathematical work of teaching, and further that this work might have specific entailments across diverse contexts of practice. These entailments are practice-based, and so require a study of mathematics in use in teaching across diverse contexts of practice. An identification and description of this work is what is needed, and embedded here is the practical problem of the mathematical preparation of teachers. And so a research agenda follows. The underlying assumption is that such mathematical foci could then be included in the mathematics preparation and ongoing professional development of teachers, and that this will make a difference to their being able to teach mathematics well. Of course, effective and appropriate mathematics in teacher education remains a further empirical question.

The beginning of this work is underway, fuelled by a brief and partial secondary analysis of some of the data collected in my first study on. Teachers' knowledge of their practices in multilingual classrooms (Adler, 2001). I thus turn in this final section of the paper to discuss two teaching episodes in the earlier study, and the mathematical work the teachers appeared to have done, or needed to do to construct and mediate the tasks they presented their learners .

❖ **working across discourses**

In a lesson preparing the ground for Grade 11 learners to study linear programming, the teacher developed a table on the chalk board with learners which focused their attention on two particularly troublesome phrases in mathematical English that learners would need to use in linear programming problems: 'at most' and 'at least'. A detailed description of this lesson episode and its context is provided in Adler (2001) and not repeated here. My attention instead is on uncovering the mathematical work entailed for. the teacher in preparing and teaching this activity.

The learners in this class were predominantly Tswana-speaking. and with varying degrees of fluency in ·English (and so too Mathematical English). Consequently, the teacher focused their attention on 'at most' and 'at least' in a way that had them relate these phrases firstly to the related mathematical phrases 'not more than' and 'not less than'. (Negations): secondly to everyday contexts where these terms might be used; and then thirdly to the symbolic forms such expressions would take. A table was constructed in interaction with learners that related the Mathematical English to everyday English and to a symbolic expression as captured below.

| Mathematical words | Settings | Mathematical symbols |
|---|---|---|
| Not more than | You can spend not more than R50 | $\leq$ |
| Not less than/ at least | There were at least 10 people at the meeting- | $\geq$ |
| At most | You can spend at most RSO | $\leq$ |

My purpose here is not to discuss how this table came to be used, or whether its construction is an appropriate and productive way to deal with these language demands in linear programming. It is rather, at this departure point in a research process, to ask: what mathematical work is in play for the teacher as she works with a diverse class of learners to access the mathematical practice of translation between verbal and symbolic mathematical expressions, and between these and everyday discourses, and all in English.

What is the range of mathematical practices entailed in the task of relating and tabulating everyday phrases, mathematical phrases and mathematical signs and symbols? Firstly, translation across discourses is obvious. In addition, 'ordering' is critical in this mathematical work. Translating the expression 'not more than R50' into a mathematical expression in symbolic form (which is what linear programming tasks require) is not only a matter of identifying an appropriate sign or symbol (£), but critically on how it is ordered as it is connected with relevant other signs and symbols - here 'x' and 'R50'.

Ordering is in play even when translating ordinary. English to mathematical English. We know too well the difficulties with translating "there are twice as many chairs as tables" into a mathematical statement. I have often heard African speakers of English ordering numbers where the larger number is placed first e.g, "I need 5 or 4 of those" - a sign that the spoken ordering of numbers in some African languages differs from such ordering in English and Mathematical English. Order matters in translation, be .it from one language to another across discourses, and from words to. Symbols in mathematics. And it matters in specific ways in mathematics. So what then is entailed in teaching ordering in mathematical translations, an aspect of mathematical work that was not actually in focus in the lesson from which this episode is drawn, nor in the analysis of the lesson in my earlier research?

Just as 'ordering' matters in the mathematical work of teaching. so do metaphors for teaching. This latter might be better categorized as pedagogic mathematical knowledge for teaching. In the above episode, the teachers· choice of contexts/settings to illuminate these notions is interesting, and deeply contextual. At that time in South Africa, secondary school learners were caught up with the politics of the demise of apartheid, and spent much of their time during school in political meetings. Metaphors matter, as they carry meanings in everyday discursive practices that can enable or obscure the mathematical notion the teacher is hoping to illuminate. Walkerdine's work (1988} as discussed in Adler (2001) is illuminating here.

This discussion on key mathematical practices in the kind of task exemplified above provides an additional lens on aspects of the language of mathematics that are well known. It has also brought out some inter- relation between the mathematical work of teaching in general, and an aspect of how this might take on specific significance in linguistically diverse settings. It was a re-examination of this earlier work that suggests that examining mathematics in use in teaching across classroom contexts would be productive.

- ❖ Designing and mediating productive tasks

I now turn to a second episode, also described in detail in Adler (2001), this time in a Grade 8 classroom where the teacher was English-speaking, but most learners had languages other. than English as their main language. Here the teacher was focused more on learners talking to learn, and learning to reason mathematically. As part of a sequence of tasks related to properties of triangles, the teacher gave the activity in the box below to her Grade 8 classes. If any of these is impossible, explain why. Otherwise draw it.

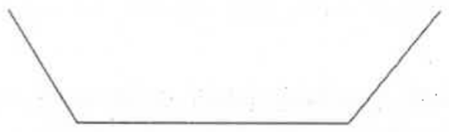
- Draw a triangle with 3 acute angles.
- Draw a triangle with 1 obtuse angle.
- Draw a triangle with 2 obtuse angles.
- Draw a triangle with l reflex angle.
- Draw a triangle with 1 right angle.

The task itself evidences different elements of important mathematical work entailed in teaching learners to reason mathematically. Firstly, this is not a 'typical' task on the properties of triangles. A more usual task would be to have learners recognize different types of triangles, defined by various sized angles in the triangle.·What the teacher has done here is recast a 'recognition' task into a 'reasoning' task. She has constn1cted the task so that learners are required to reason if they are to proceed with the task. In so doing, the teacher sets up conditions for producing mathematical reasoning in the lesson and related proficiencies in her learners.

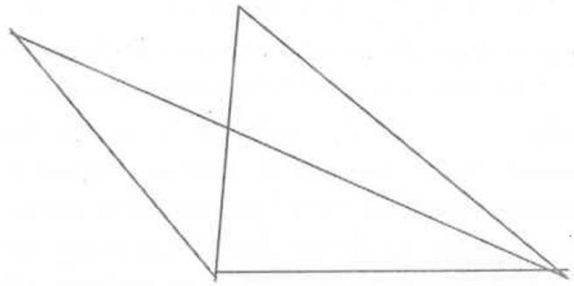
In constructing the task so that learners need to respond whether or not particular angle combinations are 'impossible' in forming a triangle, the teacher expects a proof-like justification, an argument or explanation that will hold··in all cases (for otherwise it will not be impossible). What is entailed here, mathematically? The teacher would need to think about the mathematical resources available to this classroom community with which they could construct a general answer (one that holds in all cases). The learners do know and have worked with angle sum in a triangle. What e1se might come into play as learners go about this task?

In this particular classroom, students worked on their responses in pairs. The teacher moved across the classroom, asking questions like: Explain to me what you have drawn/written here? Are you sure? Will this always be the case? I foreground here student responses to the item: Draw a triangle with two obtuse angles. On this part of the task, there was a range of learner responses - indicative of ci further skill embedded in this task. It is designed in a way that diverse learner responses are possible and enabled.

- *Some said: It is impossible to draw a triangle with two obtuse angles, because you will get a quadrilateral. And they drew:*



- Others reasoned as follows: an obtuse angle is more than 90 degrees and so two obtuse angles gives .you more than 180 degrees, and so you won't have a triangle because the angles must add up to 180 degrees
- Joe and his partner reasoned in this way: If you start with an angle say of 89 degrees, and you "stretch it", the other angles will shrink and so you won't be able to get another obtuse angle. They drew:



Now it is the teacher's task to mediate across these responses, and enable her learners to reason whether each of these responses is a general one, one that holds in all cases. The interesting interactions that follow in the class are described and problematised in Adler (2001) and will not be focused on here. In the many contexts where I have presented the study and this particular episode, much discussion is generated both in relation to the mathematical status of the responses, and their levels of generality, as well as simultaneous arguments as to what can be expected of learners at a grade 8 level. What constitutes a generalized answer at this level? Are all three responses equally general? Is Joe's response a. generalized one?

These are mathematical questions, and the kind of mathematical work this teacher did on the spot as she worked to value and validate what the learners produced. The point here is that this kind of mathematical work i.e; working to provoke, recognize and then mediate notions of proof and different kinds of justification is critical to effective teaching of 'big ideas' (like proof) in mathematics. Yet this kind of mathematics is rarely the focus of attention in the· mathematical preparation of teachers.

## IN CONCLUSION

In this paper I have presented a description of how I have come to a particular research agenda focused on the mathematical work of teaching, and its possible entailments across diverse classroom contexts; My driving motivation is: If we know more about 'what' and 'how' mathematics is used in and for teaching across contexts, we will then be able to grapple with whether, how and where these practices a.re teachable, and then too who
(What expertise) is required for this teaching?

In addition to motivating this agenda, I turned to some· secondary analysis of data from an earlier project to explore the value of studying the mathematical work of teaching across contexts. Looking inside teachers' work in two different multilingual mathematics classrooms in South Africa, it does appear as if context matters in unearthing the specificity of teachers' mathematical work. The elements of the mathematical work of teaching identified in this paper viz. translation across· discourses,. ordering in mathematical representation, recasting tasks, valuing diverse ·levels of justification, are typically not in focus in mathematics courses taken by prospective teachers in their undergraduate mathematics study in South Africa, and because they are mathematical in nature, they do not appear to be in focus in 'methods' courses either. There is thus much work to do in mathematics teacher education, both practically and in terms of research.

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Conceptual Article

# Dynamic Geometry Enriches the Design of Curriculum Materials for Middle Secondary School Mathematics

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**Abstract**

This paper examines the role of Dynamic Geometry Software (DGS) in enriching the design of curriculum materials for middle secondary school mathematics. It highlights how interactive visualisation, construction, and manipulation of geometric figures foster students' understanding of key mathematical concepts. The study emphasizes how DGS supports conjecture, exploration, and reasoning, moving beyond rote learning to deeper conceptual engagement. Design principles for integrating dynamic geometry into curriculum tasks are discussed, with a focus on enhancing student motivation, spatial reasoning, and problem-solving abilities. The authors argue that well-structured curriculum materials incorporating dynamic geometry provide valuable opportunities for both students and teachers to experience mathematics as an active, exploratory discipline.

**Keywords:** *Dynamic Geometry Software (DGS), Curriculum Design, Middle Secondary Mathematics, Geometric Reasoning, Visualization, Conjecture and Proof Mathematics Education*

## INTRODUCTION

The RITEMATHS project explored the use of technology to support the use of real world contexts to enhance the learning of mathematics in the middle secondary years. Dynamic geometry linked to real world images

.or used to create dynamic simulations provided opportunities for students· to collect real or simulated data and to gain mathematical understanding through exploration using multiple representations.

Tasks which accessed these features proved to be valuable for both increasing students' engagement and their depth of mathematical thinking. The colour. Movement and interaction created a halo effect valued by teachers for its impact on stu- dents' general attitude towards studying mathematics.

Teaching fourteen and fifteen year olds mathematics presents many challenges. Engaging these students can be difficult, especially as the mathematics they study becomes more abstract. A range of technology, now commonly available, offers opportunities to bring the real world into the mathematics classroom, to add visualisation, colour and animation not possible in a traditional classroom and to deepen the mathematical think- ing we expect in various topics of the curriculum.

The three examples of curriculum materials presented in th.is paper were developed as part of the RITEMATHS project (HREFI) and focus on some of the affordances of dynamic geometry packages. The project aimed to investigate the use of real (R) world conte)..'t problems with the assistance of Information Technology (IT) to enhance (E) middle secondary school students' engagement and achievement in mathematics (MATHS).

Increasingly mathematical software is becoming. Integrated so that dynamic geometry may be seamlessly linked to a scientific calculator, lists or spreadsheet. a function grapher and perhaps a computer algebra system. Such packages may also allow for word processing and the use of digital images. These facilities allow the teacher to:

- give high quality, attractive presentations,
- bring real world situations into the mathematics classroom,
- set tasks that allow students to explore mathematical regularities and variation within one mathematical representation,·
- Set tasks that allow students to explore mathematical ideas by linking different mathematical representations of, for example, the same function.

The facility to quickly prepare high quality. Attractive presentations is an advantage for the teacher. Such presentations not only attract students' attention but. When pre-prepared, may lessen the immediate cognitive load for the teacher as they deal with the class, the mathematics and the technology.

The value of real world problems has been well recognised (see for example Burkhardt, 1981). Linking to the real world has a cognitive impact as students learn that mathematics has both meaning and real world consequences. Such problems:

- foster general competencies and attitudes;
- prepare citizens who have critical competence;
- equip students with the skills to utilise mathematics for solving prob- lems;
- give students a rich and comprehensive picture of mathematics and
- can prompt mathematics learning of a traditional type {Blum and Niss, 1989).

In addition Pierce and Stacey (2006) report that teacher choices of real world problems enhance the image of mathematics by taking advantage of a 'halo effect'. The term 'halo effect' dates back to the l 920's work of Edward Thorndike (1920) showing that assessments of specific traits of a person or thing are markedly influenced by an overall impression, which may itself be based on little evidence. In marketing, the term 'halo effect' is used to describe the phenomenon of extending a positive view of one attribute or item to an entire brand.

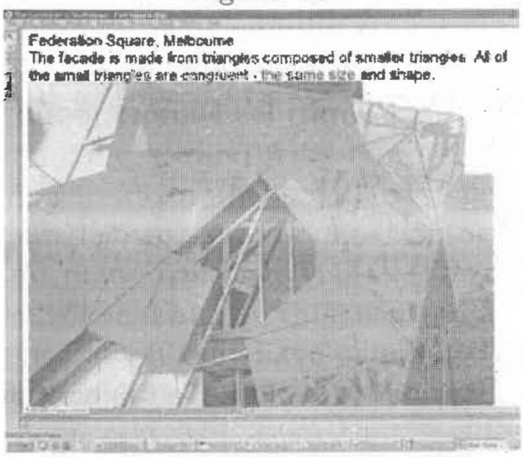
We have observed that many mathematics teachers' take advantage of this psychological strategy by associating mathematics with simple pleasures from real world contexts. They aim to capitalise on students' appreciation of colour, fun, and pleasant shared experiences to create a 'halo' effect that extends from the pleasant experience to the immediate mathematical task and beyond to the study of mathematics in general.

The real world may be brought info the mathematics classroom in two ways: through the use of digital images and through the use of simulations. Both of these can occur using dynamic geometry. In this paper the first two examples illustrate the use of digital images while the third illustrates the use of simulation. In. the sections below each example will be illustrated and discussed with attention to both the cognitive and affective aspects of student learning which are built into the design.

**Example 1: Federation square and the golden ratio**

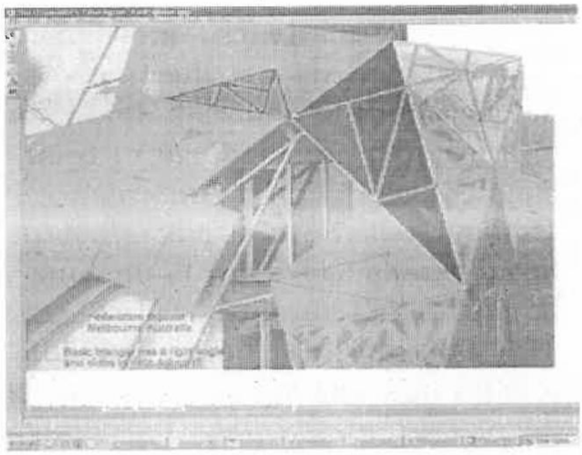
At Federation Square in the centre of the city of Melbourne there is a facade made of 22000 zinc, glass and sandstone triangles. Small triangles are progressively arranged. into larger geometrically similar triangles, using a famous pinwheel tiling. Figure 1 shows both a section of the facade and the same section with line segments used to highlight the pattern of tri- angles. All of the small triangles are congruent and all the highlighted triangles are similar. for Victorian students this is a familiar landmark that many will have visited. For many students it will evoke shared memories of looking at this feature as part of a school excursion to the city especially if they have completed the city 'maths trail' (Vincent, 2007). The mathematics is linked to the real world because the students have seen the facade and will learn that the fascinating and aesthetically pleasing at- tributes of the patterns in the facade are linked to the mathematical properties of the shapes used.

Figure 1a




THE FAÇADE OF FEDERATION SQUARE, MELBOURNE, USES A PINWHEEL TILING

Figure 1b



FAÇADE WITH THREE SIZES OF TRIANGLES HIGH-LIGHTED.

We have prepared a dynamic geometry file (in this case a Geometer's. Sketch Pad (GSP) file) that allows the user to access some of the functional opportunities afforded- by the software. It takes advantage of the facilities to insert pictures; create points, lines and polygons in front of the image; create triangles with specified mathematical properties, measure the length of lines; and perform calculations. Two sorts of constructions are used in the file: (i) triangles drawn on the photo highlight features as in Figure la (where right angles in the real world may not be right angles on the photo) and (ii) constructed right-angled triangles with side lengths that are multiples of 1, 2,v5 represent the triangles of the theoretical tiling (see Figure le). Investigation of the mathematical properties of the triangles in the facade prompts discussion of properties of triangles, measurement, congruent triangles, similar triangles, tessellations and ratio. If students know that the triangles highlighted in Figure 1b are all similar and right-angled, then it.is easy to see that the side lengths are in the ratio l:2:v5 (because the sides of a large ·triangle are made from 1 and 2 copies of the hypotenuse of the next smaller triangle).

None of these images are dynamic. What, then, does the use of dynamic geometry software in this way offer for teaching? It is true in this case that the construction, investigation and mathematical operations could be carried out by students using photocopies, pen, paper, ruler, scissors and calculator. The dynamic geometry _programs offer the opportunity to. Project the photo of the real facade and examine it, with whole class discussion, within the orderly confines of the classroom. The material shown here was used in conjunction with paper measuring. Which students could further link to the real world objects. Olivero and Robutti (2007) point out that discussion of measuring in _the real world and the digital world along with discussion of the theoretical mathematical world provides an opportunity to exploit what Pierce and Stacey call the contrast between ideal and machine mathematics. In ideal (theoretical) mathematics the length of the sides of the triangles will be exactly represented by multiples of 1, 2, and v5. When the lengths are measured using the dynamic geometry software the results will be approximate and the accuracy reported according to the number of decimal places set.

In addition although the triangles are congruent, measures for equivalent sides may slightly vary with the orientation of the triangle (a software. phenomenon), the relative position of the camera and the real walls of Federation Square, and the camera lens properties. Some of this va1iation requires discussion with the students. Further, if students have hand drawn or cut out triangles, measures with their rulers will be limited by the accuracy of the ruler. If students are to use their mathematics to solve real problems then it is vital that they learn to understand the practical constraints associated with measurement, whether it is using real world objects such as a rule or on screen measurements such as through dynamic geometry.

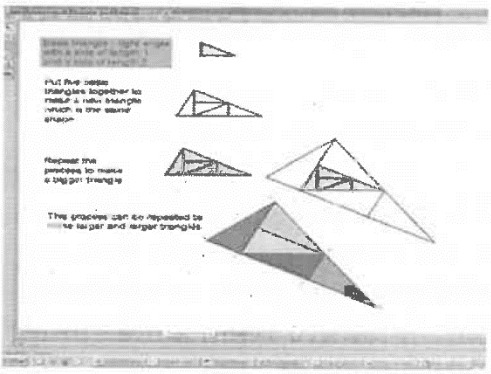


Figure 1c. Illustration of how to make the pinwheel tiling.

Dynamic geometry may be used to encourage students to develop a sense of discovery as active learners.· Azzarello et al {2002) describe the ascending and descending modalities of a lesson using dynamic geometry: ascending as the learning activity focus moves from perception of drawings to analysis through theory then descending when the focus moves from theory to drawings. This is reminiscent of the stages of mathematical modelling, as problems are transformed from problems in the real world, to problems in the mathematical world, solved and then having the solution interpreted and evaluated in real world terms. His also reminiscent of the horizontal and vertical mathematizing of RME (realistic mathematics education). Descriptive names have been given to the various ways in which the user may use dragging to explore mathematical regularity and variation. Azzarello et al (2002) name 'wandering dragging', 'bounded dragging', 'guided dragging', 'dummy locus dragging', 'line dragging', 'linked dragging' and 'dragging test'.

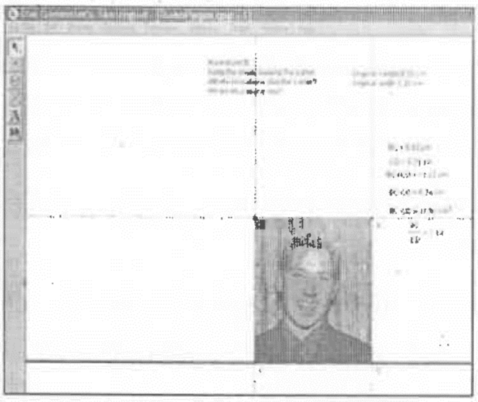
A number of these dragging modalities are illustrated by the examples which follow. It is important to note that from classroom research Azzarello et al note that the students· solution processes develop through a sequence of different modalities following an evolution from perceptive to theoretical through ascending and descending modalities. Exploratory tasks require careful didactical design to support students in such a learning process. Sinclair (2003) found value from using carefully prepared dynamic geometry files that encourage discovery without confusion and which can be delivered within a limited time frame. Such files should also be accompanied with carefully designed questions that help students "learn how to use change to explore and_ how to extend their visual interpretation skills" {p312).

**Example 2: Digital images, aspect ratio and similarity**

The world of digital cameras and resizing images is familiar to our students. So too is the variation in aspect ratio which must be considered When watching DVDs on computers, televisions or large screens. We have used these real world experiences tq introduce ideas :Of similarity using rectangles {rather than the classic approach with triangles) with "looks the same" as the fundamental meaning given to similarity. Digital images pasted into a dynamic geometry package, such as those shown in figure 2, offer links between mathematics and the real world. Students can manipulate the images, deciding whether they 'look the same' when the size changes. Through various modalities of dragging {as discussed above) students explore the regularity and variation of the length and breadth and also the _ ratios; sums and differences of these measures, while observing the changing image. When the image 'looks the same' the ratio of the length and breadth remains the same as the original. but the actual measurements and other quantities calculated from them alter. These experiences may impact on cognition in the areas of measurement, calculation, ratio, aspect ratio and similar figures. Positive affective impact may be gained by the choice of photos. These could be taken around the school or include images of the· students from the class. In this example different· dragging modalities are used to develop theory and test theory and these are structured into the design of the dynamic geometry worksheets.

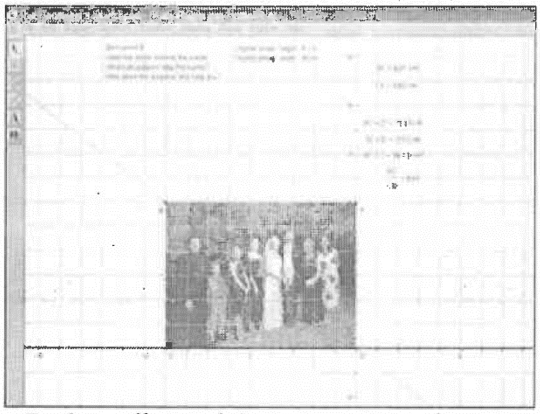
In the worksheet illustrated by figure 2a students are asked to drag the corner of the photo so that the image 'looks the same'. They are encouraged to use 'wandering dragging' and observe the effect on the shape of the image and on the lengths; ratios, sums_ and differences of the side lengths. From this experience they form conjectures about the preservation of aspect ratio. Initially, for example, students find that the images that look the same all have the same ratio of length and breadth. They also usually note that the one dragged corner stays on an (extended) diagonal of the image. Putting a trace on the dragged corner reveals this. This property can be linked then to the invariant ratios of the sides, and the equality of the factors the stretch or shrink the length and breadth. The dynamic geometry task has a series of worksheets. Figure 2b comes later in the series and provides both the grid and a diagonal line. This may either be used as a guide for students who have not developed a theory or the grid and line also make it easy for students to test their own theory for the preservation of aspect ratio. Other examples of the use of dynamic geometry and other image software are given by Pierce et al (2005).

Figure 2a



Explore effects of dragging corner of image-wandering dragging encouraged.

Figure 2b



Explore effects of dragging corner of image-bounded and guided dragging

**Example 3: The biggest volume for a box made from A4 paper**

The third and final example. requires pre-calculus students to explore an optimisation problem: what is the maximum Volume of an open box that can be made from a sheet of A4 sized paper or card? Our teachers have found that the impact of this task is greatest, and is remembered by students when they learn calculus some years later, if they each draw an appropriate net and construct a box and find its volume and plot a point on a scattergram of class results. This practical and numerical work pre- cedes the experiences with the dynamic geometry and also precedes the algebraic formulation. Greatest affective impact is gained by using brightly coloured paper, and stacking the boxes as in figure 3a thereby exhibiting their decreasing bases, and increasing height. The difficulty of judging which box has the greatest volume must be resolved using mathematics - the stacking does not help with that.

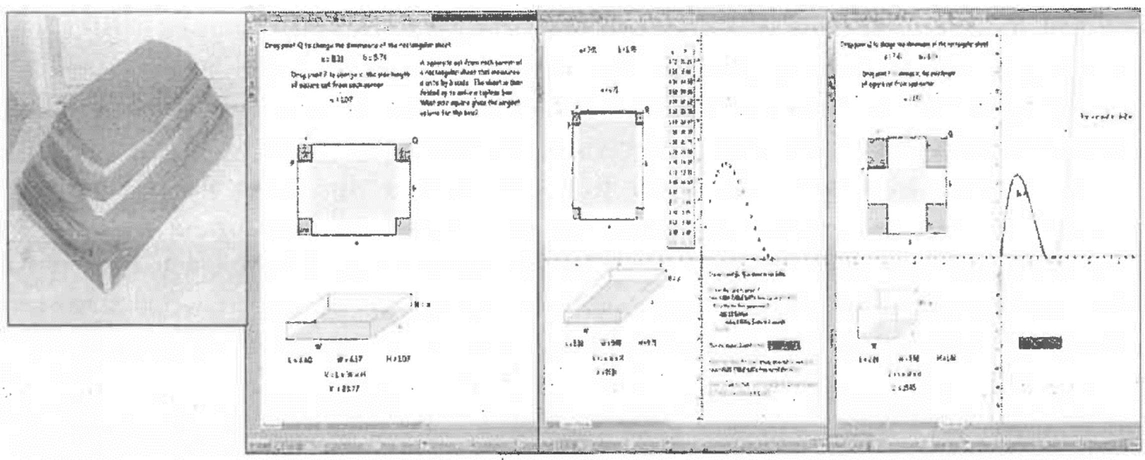


Figure 3a (far left) Coloured open boxes each-constructed from A4 paper. Figure 3b (mid-left) Dynamic file of open box and its net. Figure 3c (mid-right) Adding table of values and graph of height and volume. Figure 3d (far right) Open box, net and graph of volume function showing limited domain.

Because dynamic geometry is now linked with data capture, data graphing, list construction, its use enables students to:

- examine and explore a virtual model of the paper cutting, fully mathematised, which corresponds to their paper model but is manipulable in size;
- obtain first hand virtual experience of how the volume changes when the height of the box is altered (with the kinaesthetic of dragging);
- capture data to create a table of values of the height and volume (this step highlights empirical aspects of mathematical discovery);
- display the data graphically as isolated points, and as a curve:
- test whether formulas derived deductively match the data.

## CONCLUSION

The examples above have illustrated three different types of uses of dynamic geometry. In the first, the dynamic geometry was especially used to enhance the presentation of the material. Students could have used the traditional tools of the pen-and-paper classroom, but the computer added positive associations, colour and clarity. In our research project, we particularly noted the very strong emphasis that teachers placed on these superficial' uses of technology, simply to brighten the classroom and improve attitudes to mathematics. Our aI1alysis of this phenomenon Jed us to, the psychological literature on the halo effect (Pierce c1ud Stacey, 2006}. We now understand that an• important motivation for teachers to use· real world problems, which was previously neglected in the literature (see for example, Blum and Niss, 1989) is to associate mathematics with pleasurable parts of students' lives. Teachers hope (maybe not consciously) that using pleasant contexts, images and objects in mathematics lessons will, by association, make students feel more positively disposed towards learn- ing mathematics. These. Presentation aspects of dynamic geometry and other software may (quite literally) be superficial, hut it does not mean they are not important.

The second example, introducing ideas of similarity through aspect ratio, inherently required technology to manipulate images. Azzarello's research group had identified the roles of different types of dragging in conducting investigations, and it is interesting that several of these had been independently created for use in this unit.

In the third example, we see the expansion of multiple representations in algebra that is supported by dynamic geometry. Classic writing on multiple representations for algebra (see, for example, Kaput, 1992) has considered three representations (e.g. the numerical, symbolic and graphical representations of a quadratic function), but with dynamic geometry used to study real world problems we move seamlessly between five worlds:

- real world situation:
- dynamic geometry simulation;
- numerical representation;
- graphical representation;
- Symbolic representation.

In conclusion, dynamic geometry as a stand-alone application, and even more as part of a package of mathematical tools has much to offer for teaching mathematics through real world problems.

**Acknowledgement**

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Conceptual Article

# Understanding Teachers' Mathematical Knowledge Using Non-Typical Examples

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## Abstract

Mathematical knowledge Jor teaching is the area researched by many researchers across countries. Effective and good teaching is dependent on teachers' own understanding of mathematics is well understood in the field. At HBCSE, with experience of several years of in-service teacher education programme, we conceptualize the nature of teachers' knowledge as being composed of two major areas (teachers• content knowledge and learning pedagogic techniques), found similar to what other researchers have proposed with some cultural variations. The question which I try to address here is what are the tools available for us to develop teacher's content knowledge? I discuss one of the tool as use of non-typical examples. How do they work and bring changes in teachers' cognition? Do such examples facilitate or impede learning'? .T discuss here some examples that 1 used in teacher education workshops and try to understand the learning that occurred during interviews and discussions with the teachers, which was conducted subsequent to their written response to these examples.

**Keywords:** *Mathematical Knowledge for Teaching, Teacher Education, Content Knowledge, Pedagogical Techniques, Non-Typical Examples, Teacher Cognition, and Professional Development.*

## Introduction

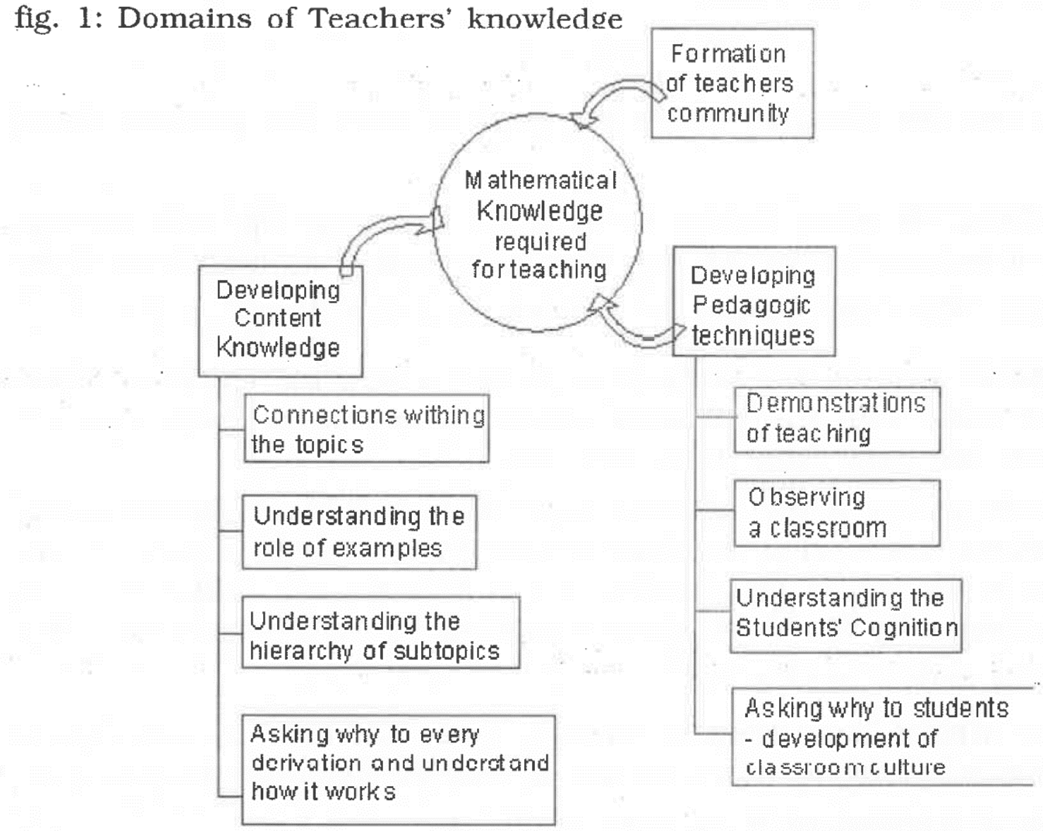
Concern about students' learning of mathematics has directed the attention of everyone towards the kind of mathematics flowing in the classroom, which in a typical Indian classroom has been originated from teacher. Hence teachers' understanding of and about mathematics becomes the crucial part of mathematical content of the classroom.

Lampert (2001) addresses that the classroom teaching proceeds simultaneously in relation with students, with content and with connection between student and content. The third part is a more complex part which demands connection between students and content, student's responses and right places for these responses for contribution to the content, progress in students' content and at the same time increase in complexity of the content in the classroom itself, and it also involves many social issues of the classroom. Observation of mathematics classes suggests that teachers' knowledge of mathematics and their ability to deploy it in teaching, matter for the quality of students' opportunities to learn (Ball. et al 2004). However, what constitutes 'knowledge of mathematics for teaching· is not commonly defined and according to me it has many parameters. The domain of teachers· knowledge identified by Shulman (1987), which he termed as 'Pedagogic Content Knowledge', made the distinction between knowing the content for 'oneself and knowing it with pedagogy required for teaching this content. This idea by Shulman focused teacher education on the content knowledge required. for teaching. This is not iri contradiction to what Dewey said that content· is not separate from its method of explanation. Better said content contains the method of explanation. But inadequate knowledge of the concepts can give rise to inadequate methods of explanation (pedagogy). Ball (2007) points out that there is much more to the" Pedagogic content knowledge" than just to refer it to a wide range of aspects of subject matter knowledge and the teaching of subject matter, but the potential of the term remains insufficiently exploited. In case ·of teachers in India it is understood that none of the educational courses give opportunity to understand the content required for their own teaching (Naik, 2008; NCTE. 2006). It seems more emphasis on methods of teaching without considering what we are going to teach. According to the Shulman's view, mathematical knowledge for teaching goes beyond that captured in measures of mathematics courses taken or basic mathematical skills. For example, teachers are not expected to . only calculate correctly but also to be able to justify each and every derivation with possible representation. How is this knowledge attained? As Ma (1999) describes Profound Understanding of Fundamental Mathematics (PUFM) is attained in Chinese teachers in their pre-teaching courses and in actual teaching careers by following the means - (a) studying teaching material intensively, (b) learning Mathematics from colleagues, (c) learning Mathematics from students and (d) learning Mathematics by doing it. So this gives us insight that, the knowledge of mathematics which is tailored to the work teacher do with curriculum materials, instruction and students is attained by doing activities pertaining to the profession of teaching. In HBCSE, we have also developed some examples to work with teachers and know more about their knowledge of mathematics. I am trying to make an attempt to see teachers thought processes in attempting such mathematical problems meant for checking their mathematical knowledge for teaching.

**Theoretical framework of teacher workshops at HBCSE**

The experience of in-service teacher education at HBCSE has led us to conceptualize teachers' knowledge as being composed of two major domains content knowledge and knowledge of pedagogic techniques (See fig. 1). Here the term 'Pedagogic techniques'. is used to explain the association between knowledge of a concept and the instruction/ demonstration required for its_ delivery considering resources available in typical classroom. The workshops we conduct for teachers talk about the techniques which are non-subjective that is independent of the individual teacher but particular to the subject of mathematics.

fig. 1: Domains of Teachers' knowledge




Even

though Figure gives the brief idea of the format of teacher education at HBCSE, it would take a long discussion to actually explain it. Let me discuss our approach to tea.cher education programmes. In the beginning of any teacher education programme we expo;e teachers to the non-typical examples. From a mathematical perspective, an example must satisfy certain mathematical conditions depending on the concept or principle it is meant to illustrate; from. a pedagogical perspective, an example needs to be presented in a way that conveys its 'message'. especially when you use it as an assessment.

Many of our textbook examples are based on using direct calculation or a direct formula. Such examples does not create any opportunities for creating conflict or challenges to what teachers know. When I say non-typical examples, these are the examples specially created for creating conflicts and to challenge one's knowledge. Another feature of these problems is they have a different format - we don't ask directly to calculate something bur give different responses as if they are· given by different students· and ask teachers to analyse them. For example a problem which we took from Ma (1999) study is as follows -

$$1\frac{3}{4} \div \frac{1}{2}$$

**state a real life situation where you need to solve the problem stated above**

Now solving 1¾ when divided by ½ is not very easy but still one can achieve it by knowing the rule that division is same as multiplication of the reciprocal. This rule changes the problem as 1 ¾ x 2 = T x 2 = T

The difficult part of this problem is to find a real life situation. Many teachers come with the problems similar to one below (with different quantities like land, milk, cloth, rice, etc). If there is a piece of land of about 1 and ¾ acres then if it divided among two brothers, what will be the share of each brother?" But see carefully the problem is not asking the division of 1 ¾ between two people but it is asking division by ½ . Think about a problem which is correct for 1 ¾ ÷ ½ . whose correct answer is 14/4.

We** at HBCSE have tried to develop collection of such non-typical mathematical problems which we use in Teacher education programmes.

**Analysis of teachers work on some problems**

Teacher education and specially assessment of teachers content knowledge requires some care which we take during adult education. Hence it puts limits on the way questions can be asked to the teachers about their own understanding of the content. We ask questions in the format which they are very familiar with. The set of examples that we give in the beginning of teacher education workshops includes four or five solutions to each of the questions given. The options are the responses given by different students, which teachers are asked to check whether each of the solution/ option is right or wrong. An example of such question is as follows-

In the following question answers given by students are given as options. Check each option whether it is right or wrong.

$7^{2/5}$ - 7 x $^{2/5}$ = ---------------

a) 0 b) $^{2/5}$ c)$4^{3/5}$ d)$^{23/5}$

When in one my recent workshop this example was tried with the middle school government teachers, 68% of teachers marked option (a) as a correct answer. But the teachers work is not over here as the format of the question demands to check each of the option given, it challenges the teachers' knowledge and beliefs about the concepts involved.

Let us see what it is that is making teachers to mark option (a). Many teachers have learned the conversion of mixed number into fractions as multiplication of the whole number and the denominator followed by addition with the numerator. This procedural understanding may develop a belief of the existence of a multiplication sign between 7 and ; . The existence is also supported by the rules from algebra as it is often said that if there is no sign between two letters (or a letter and a number) then there is a multiplication sign. So xy indicates that X x Y. Similarly.

$7^{2/5}$ indicates 7 x $^{2/5}$, Such generalisation may lead those teachers towards wrong answer that is option (a) 0.

Seeing their response, in the interview session I asked them to explain how did other students arrive to the set of answers that is (b) $^{2/5}$, (c) $4^{3/5}$ and d)$^{23/5}$. They started giving possible thinking that the student might have done. This unpacking of what students thought, gave them insight about the structure of fraction notation itself. One can interpret the notation differently and hence sometimes wrongly was recognised by some of them.

The interview with teachers showed that they knew how to carry out the multiplication of fractions or fractions with the whole numbers (numerator x numerator/ denominator x denominator). They also knew procedurally how to convert a mixed number to the fraction form. While explaining option (c) teache.rs arrived to the conflict. This conflict created the need to understand the relationship between the procedure and meaning of the procedure. The task gave them the platform to change their representation of fractions. For example, one teacher who had earlier made the error of

equating $7^{2/5}$ - 7 x $^{2/5}$ to zero, argued as follows

$7^{2/5}$ = (7 x5 + 2) / 5

$=^{37/5}$

$=^{35/5} + ^{2/5}$

= 7 + $^{2/5}$ = $7^{2/5}$

The derivation above was a rediscovery for that teacher as she proved that there is no multiplication sign in 7 ; hut 7 and ; has operation of addition in between them. For me above derivation was like giving a small arithmetic proof of 7 ; = 7 + ; . The teacher above who gave wrong answer in the beginning got the opportunity to correct herself through the help of an example and its form of presentation. Such opportunities may not be available for teachers in the traditional textbook assessment questions. Also re-teaching the concept of fraction to teachers might not create any challenges to the existing knowledge of theirs. But an example such as above gives them the platform to challenge their own understanding, repair it and reform it.

Let us see one more example -

In the following question answers given by students are given as options. Check each option whether it is right or wrong.

Find GCD of 8 and 9

| | |
|---|---|
| 72 | (21%) |
| 0 | (56%) |
| 1 | (8%) |

There is no method for this calculation (12%)

The percentage in bracket shows how many teachers marked that particular option as a correct solution (the data is again from the same set of teachers). The result is very shocking, but if one analyses it in detail we see that this . wrong understanding has emerged from some unwanted over generalisations. 72 is LCM of 8 and 9, which is hurriedly understood and lets say was mistakenly marked. What happened in option (b). following is the response from a teacher "For GCD we need common divisors, there is nothing common in 8 and 9, and nothing means zero."

Why 1 as a common divisor was missed out, may be because in unique factorization, we do not use I as factor.

This example is different from the one above, but we see the interference of the rules and language, formed for the purpose of saving time in other topics of school mathematics even in a straight forward problem like this. We teach nothing means zero, but "nothing" is with the context. So no other common divisor automatically brings us to only divisor of any number which is 1. Such numbers, which has only one common divisor that is 1 (like 8 arid 9) are called as co-prime numbers.

**Conclusion and comments**

We have many such examples, which can be used in teacher education programmes for the purpose of content development. In every teacher education programme, teachers work first individually on these examples and then discuss with me or in groups, reasons for all the solutions given for each example. The illustration above answers a question of how one can approach for development of coherent understanding of the concepts among teachers. The reteaching of any topic may not bring forward their conflicts and wrongly developed beliefs, which brings the methodology described above at the centre to any teacher education programme.

At HBCSE the group working in Mathematics Education is researching along many. parameters through which teacher development is. possible. Questions on other developmental issues such as developing pedagogic techniques, understanding coherent sequence of a topic or development of teachers community for sharing resources are welcomed.

**Note:**

**I acknowledge the role of Dr. K. Subramaniam faculty at HBCSE who had major contributions in the development of these non-typical examples.

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Research Article

# AN OBJECTIVE MEASUREMENT OF SELF-MONITORING STRATEGY ON PROBLEM SOLVING IN MATHEMATICS

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## Abstract

Approaching Mathematics through problem solving can create .a context, which simulates real life and therefore justifies Mathematics rather than treating it as an end in itself. Problem-solvers intend to employ a combination of domain-specific knowledge and strategic knowledge. Researches indicate that students' problem solving failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know. This paper describes a study that investigated the self-monitoring strategies used by high school students while working individually on a Math problem. Identifying the characteristic type of metacognitive failures of solution highlighted the distinction between the two key elements of effective monitoring: being able to recognize errors and other obstacles to progress, and being able to correct or overcome them.

**Keywords:** *Problem Solving in Mathematics, Real-Life Context, Domain-Specific Knowledge, Strategic Knowledge, Self-Monitoring Strategies, Metacognitive Failures,*

## Introduction

In review of progress in problem solving research over the past 25 years, Lester (1994) noted with some concern that research interest in this area appears to be on the decline, even though there remain many unresolved issues that deserve continued attention. One such issue highlighted by Lester was the role of self-monitoring in problem solving- where self-monitoring refers to what students know about their own thought processes, and how they regulate their thinking while working on problem solving.

Although the importance of self -monitoring is now widely acknowledged, teachers in India still lack an adequate theoretical model for explaining the mechanisms of individual self-monitoring and self-- regulation.

The study reported here examined the individual problem solving actions of a group of high school students. The results reported here provide glimpses into the mathematical thinking of individual students as well as suggesting follow up investigation into the nature of collaborative monitoring and regulation.

**Research Questions**

The specific research questions addressed in this study are as follows:

1. What strategies do students use in attempting to solve a problem?
2. How do students recognize and respond to obstacles to progress?
3. How is metacognitive self- monitoring related to problem solving outcomes?

**Method**

**<u>Subjects</u>**

Forty-two students of standard IX from Christian Girls Higher Secondary School, Tanjore were selected as sample for the present study. Tl;:l.e decision to focus on high school mathematics class rooms was prompted in part by the introduction of revised·syllabus. The traditional emphasis on memorization and basic skills has given way to arguments that students also need to develop reasoning and problem solving capacities. The incorporation of these goals into high school teachers to re-examine their conceptions of mathematics learning and teaching; hence one of the aims of the research study was to explore the implications of the problem solving for classroom practice.

**<u>Questionnaire</u>**

The self - monitoring Questionnaire elicited students' retrospective reports on the metacognitive strategies they had employed while working on a given mathematics problem. The Questionnaire is based on an instrument used with B.Ed. trainees having choices Yes/ No by Ramganesh (2003). To make the questionnaire more appropriate for standard IX students, the original version was modified by deleting, rewording. and including some items with the consultation of experts in the field of Education and Cognitive Psychology. In the present study, the questionnaire consisted of fifteen statements to which students responded by ticking boxes marked Yes, No or Unsure

The first response sheet of the questionnaire, titled "Before you started to solve problem", listed six possible strategies concerning reading and understanding the problem. The second response sheet, "As you worked on the. Method", listed five possible strategies concerning analysis and execution of a solution; while the third, "After you finished working on the problem" offered four strategies for verifying the solution.

The Self-Monitoring Questionnaire implicitly investigate .students' ability to

•• recognize and act on metacognitive warning signals. Arising during routine monitoring, which indicate the need for regulation or repair. Each questionnaire statement is identified as a generic type of metacognitive self-monitoring or self-regulatory activity, as used in the framework of Figure

1. In addition, questionnaire statements that target the "red flags" of error detection, lack of progress, and anomalous result are identified.

| **Self - Monitoring Questionnaire Item** | **Monitoring / Regulation** |
|---|---|
| *Before your started*<br>1. I read the problem more than once.<br>2. I made sure that I understood what the problem was asking me.<br>3. I tried to put the problem into my own Words.<br>4. I tried to remember whether I had worked on a problem like this before.<br>5. I identified the information that was given in the problem<br>6. I thought about different approaches I could try for solving the problem | Assess knowledge<br>Assess understanding<br>Assess understanding<br>Assess knowledge & understanding<br>Assess knowledge & understanding<br>Assess strategy appropriateness |
| *As you worked*<br>7. I checked my work step by step as I went through the problem<br>8. I made a mistake and had to redo some working.<br>9. I re-read the problem to check that I was still on track.<br>10. I asked myself whether I was getting any closer to a solution.<br>11. I had to rethink my solution method and try a different approach. | *"Red Flag": Error detection*<br>Assess strategy execution Correct error<br>"Red Flag": Lack of progress<br>Assess understanding<br>Assess progress<br>Assess strategy appropriateness<br>change strategy |
| *After you finished*<br>12. I checked my calculations to make sure they were correct.<br>13. I looked back over my solution method to check that I had done what the problem asked.<br>14. I asked myself whether my answer made sense.<br>15. I thought about different ways 1 could have solved the problem | *"Red Flag": Anomalous result*<br>Assess result for accuracy<br>Assess strategy appropriateness and execution<br>Assess result for sense<br>Assess strategy appropriateness |

Figure 1. Metacognitive strategies examined by Self-Monitoring Questionnaire.q3b

**Task**

Because the aim of the questionnaire was to gather data on self monitoring strategies rather than simply assess mathematical expertise, it was important to supply a genuine "problem" that would challenge the students and call forth processes of interest, without requiring any specialized mathematical knowledge'. These criteria were proved . to be satisfied by the problem on simultaneous equations as follows:

**Solve**

$$3x + y + z = 3 \text{ ———————— } 1$$
$$2x + 2y + 5z = -1 \text{ ———————— } 2$$
$$x - 3y - 4z = 2 \text{ ———————— } 3$$

Students are made to recall their previous knowledge about solving linear equations having one variable and two variables. In general, they are expected to understand that anything which is unknown (solving simultaneous equations having three variables) can be found out by recalling the known rules {solving linear equations with two variables or one variable)(Advanced organizer).

Now students are expected to acquire concretized knowledge .and initial prediction about the problem. They are also confident of proceeding the problem.

Teachers presents the diagrammatic scheme of the problem as follows. (Problem solving strategy)

Strategy #1: ***Random selection :***

Solving (1) & (2)

or

(1) & (3) Reduce a -------> take ------> (4)

or variable

(2) & (3)

Strategy #2: ***Formulation of Reducing a variable***

Solve two of the equations not taken already ----> Reduce a variable which is already reduced in rule (1) -----> take-----> (5)

Strategy #3: ***Finding the value of a variable***

Solve (4) & (5) -----> Reduce a variable -----> Find the value of a variable

Strategy #4: ***Substitution***

Substitute the values of two variables in either (1) (2) or (3) whichever have small coefficients -----> Find the value of the third variable

Strategy #5: ***Logical Reasoning***

List the value of all the variables

To bring awareness of one's attention the following questions are asked;

i. What do you understand to solve these simultaneous equations?
ii. Do you understand how to proceed?

They are motivated to respond and they attend the stimuli of the task

Students are instructed to use the same strategy they use for solving linear equations having two variables, about doing Strategy #1. So students are guided to choose the appropriate strategy which they know already.

(1) x 2 => $6x + 2y + 2z = 6$

(2) => $2x + 2y + 5z = -1$

$4x - 3z = 7$…………..(4)

To bring one's awareness and self-control of his understanding which variable is to be reduced to get equation (5) by taking (2) and (3) or (1) x (3) (probing) Students are expected to develop logical reasoning and they respond the question.

(l)X 3 $9x+3y+3z = 9$

$X - 3y - 4z = 2$

$10x - z = 11$…………(5)

Now, students are given verbal direction to thinking (problem solving strategy)to proceed next;

In Strategy #3. Students are to solve for equation (4) and (5). Also they are given orientation to select the ongoing experience (orchestration) to do the task Strategy #3, as to reduce another variable.

$$(4) \Rightarrow 4x - 3z = 7$$
$$(2) \Rightarrow 30x - 3z = 33$$
----------------------------------------
$$-26x = -26$$
$$x=l$$

Now

Substitute x=1 in (4)

$$4-3z = 7$$
$$-3z = 3$$
$$z = -1$$

Students are helped to regulate their thinking (self-regulation) about how the variables have been reduced as .variables into two and two variables into one.

Now they learn to determine the order of steps to be taken to complete the task (organizing) and also the speed at which they should work this type of problem (self- regulation).

It was anticipated that students would attempt a combined algebraic/ trial and error solution. A skilled formal approach would resemble that shown above.

Students were given the written problem statement and allowed twenty to thirty minutes for working. They · were instructed to show all their working and to cross out, rather than erase, any working which was incorrect. • Only at the end of this time was the questionnaire administered, to avoid cueing students on the strategies it listed.

**Questionnaire Responses**

A high rate of Yes responses was recorded for almost all Self- Monitoring Questionnaire statements referring to metacognitive strategies. Response rates for the four statements that might prompt initial recognition o( the metacognitive "warning" described earlier (lack of progress, error detection, anomalous result - see Figure 1) are shown in Table 1. While these results seem to suggest that students were immersed in metacognitive activity. it is unwise to accept self-reports of this kind at face value as information relating to regulation of cognition is not necessarily stable (Brown, Bransford, Ferrara & Campione. 1983). The students' questionnaire responses therefore must be interpreted in the light of their actual problem solving behaviour.

In the next section students' written solution attempts are examined and, where necessary, compared with their responses to the questionnaire statements in Table 1 to reveal self monitoring successes and failures.

**Table 1: Questionnaire Responses to Metacognitive "Red Flag" Statements**

| "Red Flag" | Questionnaire Statement | Percentage of Students Responding *Yes* |
|---|---|---|
| Lack of progress | I asked myself whether I was getting any closer to a solution. | 81% |
| Error detection | I checked my work step by step as I went through the problem | 63% |
| Anomalous result | I checked my calculations to make sure they were correct | 77% |
| | I asked myself whether my answer made sense. | 84% |

**Successful Self-Monitoring**

Successful self-monitoring is difficult to detect if it merely confirms that satisfactory progress is being made. However, the students' written work did provide evidence of self-monitoring where difficulties or errors forced a change of strategy. For example, although Table l shows that eight of the fifteen students who used a mean value strategy nianaged to find one of the answers to the problem, it does not reveal that six of these students began working with a different strategy that was subsequently abandoned. In three of these cases, the change of strategy was caused by lack of progress in formulating the problem algebraically. The remaining students discarded their initial strategy because it produced an answer that was either unreasonable or inaccurate. There was also some evidence that other students rejected unreasonable answers. but were unable to identify an alternative strategy.

## Failures in Self-Monitoring

Examination of Table 1 shows that there were three broad groupings of solution strategies and outcomes:

1. Inappropriate strategies that gave incorrect answers.
2. Inefficient strategies through which it was possible, with luck and persistence, to find one answer, but that were equally likely to result in no answer being found at all.
3. Appropriate strategies that had the potential to produce one or both answers, provided that the strategies were correctly executed and a way was found to solve equations with two unknowns.

Analysis of individual students' solution scripts and questionnaire responses showed that the above strategy and outcome groupings were associated with corresponding failures to recognize, or act on, the metacognitive "red flags" described earlier:

1. Anomalous results were verified and accepted
2. Lack of progress towards obtaining an answer did not lead to a change of strategy.
3. Errors in strategy execution remained undetected.

Each of these failures in self-monitoring is described in more detail Table 2.

**Table 2 : Evidence of Self-Monitoring in Users of Inappropriate Strategies· (Incorrect Answer)**

| *Evidence from Written Work* | Evidence from Questionnaire | | | | | | |
|---|---|---|---|---|---|---|---|
| | Checked Calculations | | | Checked Sense | | | |
| | Yes | No | Unsure | Yes | No | Unsure | Total |
| No evidence of verification | 2 | 1 | 0 | 3 | 0 | 0 | 3 |
| Faulty verification procedure | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| Verified non-integral | 4 | 0 | 0 | 2 | 0 | 2 | 4 |
| Verified integral | 4 | 1 | 1 | 4 | 1 | 1 | 6 |
| Total | 11 | 2 | 1 | 10 | 1 | 3 | 14 |

**Note :** Sixteen of the total of 42 students were categorized as using inappropriate strategies: Of these, fourteen obtained an incorrect answer. (A further two students obtained no answer).

Of the sixteen students who used an incorrect formulation or assumed, fourteen obtained incorrect answers, that is, an answer that violated the problem conditions. Since an incorrect answer represents a metacognitive warning signal that should trigger a review of both the accuracy of calculations and the appropriateness of the str: c1tegy. ."t is tempting tv assume that these students did not try to verify their answer. However.evidence from their questionnaire responses and written work, summarized in Table 2, suggests otherwise; Eleven students claimed that they checked their calculations, and ten reported that they asked themselves whether their answer made sense (Table 2-Evidence from Questionnaire). In most cases, their written work confirmed that they did indeed carry out some kind of verification procedure (Table 2-Evidence from Written Work); however, many appeared to accept either an integral answer that did not satisfy the problem's explicit conditions, or a non- integral answer that did not make sense.

**Table 3 : Evidence of Self - Monitoring in Appropriate Strategy Users (Incorrect Answer)**

| Evidence from Questionnaire | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Checked working | | | Checked Calculations | | | Checked Sense | | | |
| ***Evidence from Written Work*** | **Yes** | **No** | **Unsure** | **Yes** | **No** | **Unsure** | **Yes** | **No** | **Unsure** | **Total** |
| Incorrect answer caused by undetected errors | 1 | 1 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 2 |

Note : Nine of the total of 42 students were categorized as using appropriate strategies.

Of these, two obtained an incorrect answer. (A further two obtained no answer, and five obtained one or both answers).

Nine students used algebraic or verbal reasoning strategies. Five were at least partly successful, obtaining one or both answers. and another might have found an answer if she had taken more time in systematically trialing x and y values. The other student who failed to obtain an answer was hindered by her persistent, _and fruitless, attempts to eliminate one of the two variables from the equation she had derived. Interestingly. this student stated that she was "unsure" whether she had assessed her progress towards a solution (questionnaire response). Despite using an appropriate strategy, a further two students obtained incorrect answers. Evidence from their written work and questionnaire responses is summarized in Table 3. Both these students recognized treat their answers were incorrect and / or unreasonable, but they failed to detect simple algebraic errors either while they were working on the problem or later when they checked their calculations.

**Conclusion and Implications**

The aim of the study reported here was to investigate the metacognitive self- monitoring strategies used by secondary school students while working individually on a mathematic "problem" (i.e. a task that presented obstacles to their progress). Students' self- monitoring activity was inferred from their written work on the problem and questionnaire responses. Examination of the questionnaire responses and written working of the students whoattempted the problem revealed connections between solution strategies, outcomes and self-monitoring. Analysis centred on identifying students' recognition of three metacognitive warning signals: lack of progress, error detection, and anomalous result.· Ideally, each should prompt a reassessment of either the appropriateness of the chosen strategy, or the manner in which it was executed. Thus, expected recognition and response patterns are as follows:

1. Students using inappropriate strategies •that lead• to incorrect or unreasonable answers should check their calculations for errors and. if none are found, consider a change of strategy;
2. Students using appropriate strategies that, nevertheless. Produce an incorrect answer should find and correct their errors.

In practice, only five students used appropriate strategies (algebraic or verbal reasoning, supplemented by principled testing · of values for one variable) leading to one or both answers being obtained.

Although there were instance of successful self - monitoring, it was found that many students were either oblivious to the warning signals mentioned above, or were unable to act appropriately if the signals were detected. Even if students do review their progress towards the goal, check their calculations while they work,· and attempt to verify the accuracy and sense of their answer, their worthy metacognitive intentions will be foiled if they are unable to recognize when they are stuck. have no alternative strategy available, cannot find their error (or cannot fix it if they do find it). or fail to recognize nonsensical answers;

The problem of recognizing difficulties is clearly illustrated in the work of students who "verified" answers that either contradicted the information given in the problem or made no sense in real world terms. Although it is possible that these students had misgivings that they did not record, one wonders whether their years of schooling have engendered a belief that school mathematics tasks need not make sense. Ironically, some of the students who did explicitly reject these kinds of answers could not think of any other way to attack the problem. If teachers wish to encourage students to monitor and regulate their mathematical thinking, it is important to ensure not only that they are attuned to the signals that alert them. to danger, but also that they are well equipped to :respond.

In interpreting these results. Teachers should not lose sight of the fact that the problem was chosen for use in this study because of its challenging nature - that is, it was hoped that the task would raise the types of obstacles referred to above so that metacognitive strategies would be called into play. Perhaps, then, it is not surprising that so few students succeeded in obtaining a complete solution, or in effective monitoring and regulation their problem solving activity. If fact, we have observed similar results with pre- service teacher education students and practicing teachers who have tackled this task in professional development workshops. Teachers deserve many such opportunities to analyse their own mathematical thinking and consider implications for classroom practice if they are to successfully implement current curriculum policies promoting reform in mathematics education.

From·. a methodological perspective, it is acknowledged that questionnaires should be used with due control in isolation to investigate metacognitive strategy use. For example, the findings reported here deserve follow up via individual student interviews that probe questionnaire responses and seek explanation of the solution strategies they adopted. Further research is also needed to investigate the strategies students apply in regular classroom settings, when peers become an additional resource for tackling obstacles to problem solving progress. Such a research activity should also consider implications for teaching in particular, how mathematics teachers can develop metacognitive abilities in their students. The Self-Monitoring Questionnaire, when used in conjunction with a suitably challenging task, is a pedagogical tool that teachers could use with their own classes to extend students' repertoire of metacognitive strategies, and to understand the revolution of problem action.

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Research Article

# METACOGNITION AND ACHIEVEMENT IN COMPUTER SCIENCE OF DEGREE STUDENTS

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**Abstract**

This paper reports on the metacognition and academic achievement of computer science students. The sample consisted of 59 B.Sc comptuter science students. A scale on Metacognition was used to get the data from the students. Percentage analysis, Pearson-Product moment correlation co-efficient. T-test, F-test and chi-square tests were used for analysing the data. The result shows that among the sample. There is a low negative correlation between the metacognition and achievement in computer science of degree students. Further, Female students have better metacognition than the male students, Government aided college students have better metacognition than the government college students, and students studying in women's college have better metacognition them the co-education college students.

**Keywords:** *Metacognition, Academic Achievement, Computer Science Students, Correlation Analysis, Gender Differences, Institutional Differences, and Higher Education.*

## Introduction

Nowadays, metacognition is recognized as an important mediating variable for learning. Metacognitive knowledge was defined as the knowledge one has about the interplay between personal characteristics. task characteristics and the available strategies in a learning situation (I3rown. 1978, 1987; Flavell, 1987). Declarative metacognitive knowledge was found to be 'what' is known about the world and the influencing factors of human thinking (Jacobs & Paris. 1987, as cited in Marzano et al., 1988). Procedural metacognitive knowledge deals with the knowledge of 'how· skills work and how they are to be applied (Jacobs & Paris, 1987. as cited in Marzano et al., 1988).

For successful learning learners cultivate a repertoire of metacognitive strategies that they apply when and as required by Different learning circumstances. Success hinges on the appropriate transfer of relevant strategies. Metacognitive strategy must take into account this transfer. Knowledge and control are the two consistent themes in metacognition. It involves knowledge and control of self and control of process respectively (Pa1is and Winograd as cited in Marzano et al., 1988).

In Knowledge and Control of Self, successful students are aware of, monitor, and control their learning. Central to this knowledge of self and self-- regulation are commitment, attitudes, and attention. Metacognition is at work in students who choose to commit themselves to tasks. Paris and Cross (1983) align "skill with will" which alone shows the level of commitment towards the successful completion of any task. According to Edward de Bono (as cited in Maclure and Davies, 1991) "Unless you know everything, what you need is thinking", which alone paves way to successful learning'. For successful learning and academic excellence knowledge of metacognitive strategies alone is not enough, it also depends on students being able to effectively control and monitors their learning. This in turn is influenced by the elements of metacognition, some of which are Metamemory, Meta comprehension, Self-regulation and Schema Training. Following are the some of the research studies done in metacognition: Annemie Desoete(2007), studied on evaluating and improving the mathematics teaching-learning process through metacognition and his study reveals that metacognitive skills were found to be trainable and students could learn to adopt a more orienting and self-judging learning approach. Sarah Schwarm et al.(2007)., studied on 'Using Classroom Assessment to Detect Students' Misunderstanding and Promote Metacognitive Thinking and the study reveals that instructor's perceptions of students' understanding changed through the use of Classroom Assessment Techniques(CATs), while CATs encouraged students to become more aware of their own learning. Christian et al. (2004) studied Metacognitive knowledge of writing: Students and individual differences. This study highlights the complex issue of metacognitive knowledge of writing and of using such information to develop responsive instruction in writing for students with developmental disabilities and learning difficulties in inclusive classrooms. Giuseppe et al. (2006) studied on Surfing Hypertexts with a Metacognition Tool and the result reveals that the students in the experimental group improved their comprehension of the intrinsic structure of the hypermedia and created more accurate conceptual maps. Further, the students considered the system to be a useful self-monitoring tool. The trend analysis in this area shows that only a few studies have been done. Therefore, 'Metacognition and Achievement in Computer science of degree students' is been taken up for study.

**Objectives of the study**

1. To find out the level of meta-cognition among· computer science degree students.

2. To find out the level of academic achievement in computer science of degree .students
3. To find out the relationship between metacognition and academic achievement of computer science degree students

The above said objectives are achieved in terms of demographic variables: gender, type of the college, nature of the college, educational qualification of the parents and occupation of the parents

Null Hypotheses

There is no significant difference in meta-cognition of computer science egree students with respect to their

Gender

Type of the college

Nature of the college

l.2. There is no significant association in meta-cognition of computer science degree students with respect to their

Educational Qualification of Parents

Occupation of Parents

There is no significant difference in the academic achievement of computer science degree students with respect to their

Gender

Type of the college

Nature of the college

There is no significant association in the academic achievement of computer science degree students with respect to their

i. Educational Qualification of Parents

ii. Occupation of Parents

3.1. There is no significant influence of meta-cognition on academic achievement of computer science degree students.

**Method:** Survey method of research was adopted for the study.

**Sample:** Randomly selected 59 first year computer science students from Rani Anna College of Arts and Science, Tirunelveli and St. John's College of Arts and Science, Palayamkottai were selected for the study.

**Tool:** Metacognition scale. Developed by Annaraja(2007) was used for data collection.

**Data Analysis:** Percentage.. t- test, F- test, Chi square test and Karl Pearson product moment co-efficient of correlation •were used for analysing the data.

**Table 1: Metacognition and Achievement in Computer Science Degree Students**

| **S.No** | **Meta Cognition Level** | **No. of Students** | % | **Academic Achievement** | **No. of Students** | % |
|---|---|---|---|---|---|---|
| 1. | High | 18 | ***30.5*** | High | 8 | ***13.55*** |
| 2 | Moderate | 27 | ***45.76*** | Moderate | 38 | ***64.40*** |
| 3. | Low | 14 | ***23.72*** | Low | 13 | ***22.00*** |
| 4 | Total | 59 | ***100*** | Total | 59 | ***100*** |

It is inferred from the above table that 30.5% of computer science students have high meta-cognition, 45.76% of them have moderate meta-cognition and 23. 72% of them have low level of meta-cognition.

Further, it is inferred that 13; 55% students have high Academic Achievement, 64.40% students of them have moderate and 22% of them have low level of academic achievement in computer science.

**Table 2: Difference in Metacognition of Computer Science Degree Students**

| ***Factor*** | | ***N*** | ***Mean*** | ***S.D*** | ***t-Value*** | ***df*** | ***Remark**** |
|---|---|---|---|---|---|---|---|
| Gender | Male | 17 | 27.88 | 4.314 | 3.407 | 58 | **s** |
| | Female | **42** | 32.02 | 4.009 | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Type of College | Co-education | **32** | 33.59 | 2.394 | 6.652 | 58 | **s** |
| | Women | 27 | 27.56 | 4.173 | | | |
| Nature of College | Govt. | 32 | 33.59 | 2.394 | 6.652 | 58 | **s** |
| | Govt. Aided | 27 | 27.56 | 4.173 | | | |

* Significant at 0.05 level of t' value is 2.02

It is inferred from the above table that the calculated "t" values (3.407, 6.652) are greater than the table values of "t" (2.02) for 58 degrees of freedom. Hence the null hypotheses are rejected. Thus, there is a significant difference between male and female government aided and government college students, women's and co-edification college students in their meta-cognition..

**Table 3: Association between Metacognition of Degree Students and Educational Qualification and Occupation of their Parents**

| **Factors** | | **Meta Cognition** | | | | **df** | **Calculated chi-square Value** | **Re marks** |
|---|---|---|---|---|---|---|---|---|
| | | *Low* | *Moderate* | *High* | *Total* | | | |
| Educational Qualification of Parents | Illiterate | 4 | 10 | 7 | 21 | 4 | 4.942 | NS |
| | School Education | 6 | 15 | 10 | 31 | | | |
| | College Education | 4 | 2 | 1 | 7 | | | |
| **Total** | | **14** | **27** | **18** | **59** | | | |
| Occupation of parents | Coolie | 15 | 16 | 7 | 38 | 4 | 4.545 | NS |
| | Government Employee | 2 | 7 | 5 | 14 | | | |
| | Business | 1 | 4 | 2 | 7 | | | |
| **Total** | | **18** | **27** | **14** | **59** | | | |

*Significant at 0.05 level of c2 value is 9.488

It is inferred from the above table that the calculated 'x2' values (4.942 and 4.545) are less than the table value of 'x2' (9.488). Hence the null hypotheses are accepted. Thus there is no significant association between the Educational Qualification of the parents. Occupation of the parents and metacognition of computer science students.

**Table 4: Difference !n Achievement of Computer Science Students**

| ***Factor*** | | ***N*** | **Mean** | ***S.D*** | ***t' value*** | ***Remark**** |
|---|---|---|---|---|---|---|
| Gender | Male | 17 | 5.88 | 2.088 | **0.593** | Not Significant |
| | Female | 42 | 5.52 | 2.144 | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Type of College | Co-education | 32 | 67.06 | 8.791 | 0.147 | Not Significant |
| | Women | 27 | 66.73 | 8.427 | | |
| Nature of College | Govt. | 32 | 67.06 | 8.791 | **0.147** | Not Significant |
| | Govt. Aided | 27 | 66.73 | 8.427 | | |

*Significant at 0.05 level of' value is 2.02

It is inferred from the above table that the calculated "t" values (0.593, 0.147, and 0.147) are less than the table values of "t" (2.02). Hence the null hypotheses are accepted. Thus, there is 1io significant difference between male and female students, government aided and government college students, women's and co-education college students' achievement in computer science.

**Table 5: Association between Academic Achievement and Educational Qualification and Occupation of their parents**

| **Factors** | | **Academic Achievement** | | | | **df** | **Calculated chi-square Value** | **Re marks*** |
|---|---|---|---|---|---|---|---|---|
| | | *Low* | *Moderate* | *High* | *Total* | | | |
| *Educational Qualification of Parents* | Illiterate | 4 | 13 | 4 | 21 | 4 | 1.364 | NS |
| | School Education | 8 | 20 | 3 | 31 | | | |
| | College Education | 1 | 5 | 1 | 7 | | | |
| **Total** | | 13 | 38. | 8 | 59 | | | |
| *Occupation of parents* | Coolie | 8 | '.26 | 4 | 38 | | | |
| | Government Employee | 5 | 7 | 2 | 14 | | | |
| | Business | 0 | 5 | 2 | 7 | 4 | 4.775 | |
| **Total** | | 13 | 38 | 8 | 59 | | | NS |

*Significant at 0.05 level of c2 value is 9.488

It is inferred from the above table that the calculated 'x,2' values (1.364 and 4.775) are less than the table value of·'x,2' (9.488). Hence the null hypotheses are accepted. Thus there is no significant association between the educational qualification of their parents, occupation of their parents and achievement in computer science of the degree students.

**Table 6: Correlation between Meta Cognition and Academic Achievement Computer Science Degree Students**

| | | | *Remarks** |
|---|---|---|---|
| Meta cognition and Academic Achievement | df=57 | r= -0.187 | **NS** |

* Significant at 0.05 levels is 0.250

It is inferred from the above table that the calculated 'r' value 0.168 is less than the table value of "r" (0.250). Hence the null hypothesis is accepted. Thus, there is no significant correlation between the brain dominance and academic achievement of computer science students.

**Findings and Interpretations**

1.1. The percentage of computer science students having high level o.f meta-cognition is 30.5;; 45.76% students have moderate level of meta -cognition

1.2. There is a significant difference between male and female computer science degree students in their meta-cognition. However, while comparing the mean values, female students (27.88) have better metacognition than the male students (32.02).

1.3. There is a significant difference between government aided and government college computer science degree students in their metacognition. However, while comparing the mean values, the government college students (33.59) have better greater level of meta-cognition than the government aided college students (27.56).

1.4. There is a significant difference between women's and co-education college computer science degree students in their meta-cognition. However, while comparing the mean values, the students studying in women's college (33.59) have better meta-cognition than the students studying in co-education colleges (27.56).

1.5. There is no significant association between the educational qualification of their parents, and meta-cognition of computer science degree students.

1.6. There is no significant association between occupation of parents and meta-cognition of computer science degree students.

2.1. 78.95% of students have high level of achievement in computer science.

2.2.There is no significant difference between male and female students in their achievement in computer science.

2.3.There is no significant difference between government and government aided college students' achievement in computer science.

2.4.There is no significant difference between the students from Women's and Co-education colleges in their achievement in computer science.

2.5. There is no significant association between academic achievement in computer science and educational qualification of the parents.

2.6. There is no significant association between academic achievement in computer science and Occupation of the parents.

3.1. There is no significant influence of meta-cognition on academic achievement in computer science of degree students.

**Suggestions**

We suggest the following activities to improve the meta-cognition of the computer science students

I. Since every individual has different levels of meta-cognition, the teachers in the colleges can emphasize on varied teaching or learning goals, and thus different activities can be derived for apparently the same educational tasks.

II. Opportunities should be given to the students, to plan appropriate activities and observing activities engaged by others with different values or socio-cultural backgrounds which will enable the students to reflect on their own goals.

III. Teachers can use c01nputers to improve their meta-cognition by giving them. A set of expe1iences with specific and rect1rrent events where personal decision making is required. Opportunities to appreciate what other sources of information are important to consider and to reflect on. This kind of meta-cognition is useful because, in many situations, especially in complex teaching situations, teachers often lack background information to know what solution can be sought and which strategies will work.

IV. Teachers must explain to learners about why, when, and how to use metacognitive strategies for successful academic achievement.

V. Intelligent Tutorial System and Blended Instruction can be adopted to foster the metacognitive strategies for learning among students.

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Conceptual Article

# Why students find Mathematics difficult?

*S Sundaram

## Abstract

This paper explores the reasons why students often find mathematics difficult at the school level, with a focus on primary education where foundational attitudes toward the subject are formed. The study identifies two core issues: the widespread misunderstanding of mathematics among teachers, and the inappropriate pedagogical methods used in classrooms. Mathematics is frequently misinterpreted as a subject of rote memorization rather than a way of understanding patterns, relationships, and abstract concepts. Teachers' inability to convey conceptual clarity, combined with linguistic and pedagogical limitations, contributes to students' difficulties in learning. Additional challenges include inadequate understanding of children's cognitive development, overemphasis on writing, and limited use of activities and visualization. The paper suggests classroom strategies such as relating concepts to students' experiences, employing concrete-to-abstract progression, encouraging peer interactions, using activity-based methods, reducing reliance on rote memorization, and promoting meaningful practice over mechanical drills. These approaches aim to make mathematics learning accessible, enjoyable, and conceptually rich.

**Keywords:** *Mathematics Learning Difficulties; Primary Education; Misunderstanding of Mathematics; Inappropriate Pedagogy; Conceptual Understanding; Activity-Based Learning; Student Engagement.*

There are two reasons why students find learning of Mathematics difficult at school level. The initial likes and dislikes for a subject are sown in the Primary school. Hence in this article we will restrict ourselves to the issues mainly dominant in Primary School. In the course of the article we will also see why the emotional impact of a subject in Primary School is more important in mathematics than in other subjects.

The first reason is the misunderstanding of Mathematics as a subject, among teachers themselves. Teachers themselves have been victims of an edu-cation system which has always misunderstood Mathematics. Further, teach- ers do not read anything in Mathematics apart from their textbooks. Hence they are totally ignorant of the true nature and beauty of Mathematics.

The second reason flows from the first. Because of a lack of under- standing of the true nature and beauty of mathematics, teachers teach the subject in class in very inappropriate ways. As a result the students are. Unable to understand the subject and they start disliking it. Naturally they find it very difficult. This entire process perpetuates the myth that Math- ematics is a naturally difficult subject, given to only a few gifted individuals to comprehend. This myth has become so much a part of our social lore that the only subject people feel proud of publicly announcing their ignorance in is Mathematics. It is almost like badge of honour.

Hence the root cause is the misunderstanding of Mathematics by the society in general and teachers in particular.

**Misunderstanding Mathematics**

Firstly Mathematics is not a subject as the word is understood in school. A subject, as understood in school, is full of content which the student has to master and this mostly implies memorizing: Mathematics has very little con- tent to memorise.

Mathematics is a way of looking at the world around us, discovering the patterns and relationships that we see, and evolving a language to express these patterns and relationships. The final language in which these relationships & patterns are expressed, which is numbers and figures & symbols, will not make much sense unless the underlying thought processes have been experienced at least to some extent.

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Teachers consider the numbers and symbols as the content to be re- membered and fail to enable students to see the real concepts behind them.

A simple example will make this point clear. In teaching 2 digit numbers like 57, teachers teach that 5 is in the ten's place and 7 is in the one's place. Students also repeat these sentences mechanically and teachers are convinced that the students have understood Place Value. But, when probed a little further, most teachers are unable to answer the following questions.

a. What is the meaning of ten's place? Why ten's and not nine's place?
b. Can there be a six's place similar to a ten's place?
c. Why is ten important?
d. Why are there only ten numerical symbols 0,1,2,3,4,5,6, 7,8,9?
e. Why is ten written with a 1 and a zero and not with a different symbol?
f. What is the difference in the meaning of ten and ten's place?
g. Can you draw a picture to explain what you really mean?
h. Do students of class 1, where Place Value is normally taught, really understand the difference between all these words (ten, one, ten's, one's, place) that are used by the teacher?

Hence Mathematics is full of concepts hidden behind numbers and symbols. Teachers tend to focus on these symbols without clarifying the concepts underlying them.

Secondly all these concepts are organized in a hierarchical manner. Understanding of more complex concepts depends on understanding the underlying simpler concepts. Hence Mathematics is like a house of cards with a broad base and a pyramid like structure which narrows as it goes upwards. If any of the lower cards are shaky, the entire structure will wobble. Most of these basic concepts are abstracted from real life situations. But as they become more and more complex, a stage comes where the relation to physical reality becomes more and more tenuous.

Let us take numbers as an example. Numbers start with Natural numbers which are used for counting any collection of discrete objects. Then we add Zero to it. Most teachers do not even realize that our ancestors argued for thousands of years to decide whether Zero is a number or not? Then fractions were added to the collection of numbers.

Do we even realize that an apparently simple fraction like ½ is totally different from a number like 2? We can directly see 2 in real life in terms of our legs or hands or legs of a bird or a pair of birds. But we cannot see ½ directly. No physical thing is ½ in isolation. It is ½ only in relation to some- thing else. It has no independent existence in the real world. But the conceptual world of Mathematics, accepts½ as a number equal in status to 2. Hence 2 and ½ are different in their real world status but accorded the same status in mathematics.

This fundamental fact is one of the reasons children find it difficult to understand the concept of a fraction. Instead of taking time to allow them to absorb this new idea, teachers are in a hurry to make them write fractions in their notebooks and memorise words like Numerator, Denominator, Proper, Improper etc etc thus confounding the confusion.

**Inappropriate Pedagogy**

The problem of misunderstood Mathematics is compounded by inappropriate pedagogy. First is the language through which Mathematics is taught. In most other school subjects, like social studies or science, some day-to-day language like English or Hindi is used to convey the content of the subject. But languages are inadequate to completely convey the meaning of Mathematical concepts. The problem becomes severe in Primary School since the language skills of students themselves are not fully developed. Hence they struggle to understand and express mathematical concepts through language.

Please try the following experiment even with experienced Mathematics teachers. Draw a very simple geometrical figure like a Circle and Square intersecting each other on the board. Ask one of the teachers to describe the figure and make another teacher draw it only by hearing the oral instructions, without seeing the drawing. Invariably they will get it all wrong thus exposing the inadequacy of language to express mathematical concepts be- yond a point. This points to the necessity of developing a precise mathematical vocabulary and understanding its meaning correctly.

Another related problem is that many words used in school mathematics have different meanings in day-to-day life. Examples are problem, rest, left, balance, proper, improper, principal, interest etc etc. When these words are used in the mathematics class without adequately bringing out the difference in the meaning, children find it difficult to understand them. There also are many words in Mathematics, which are never used in day-to-day life. Examples are words like Numerator, Denominator, Dividend, Divisor, Quotient etc. When stu- dents are expected to remember these words at too young an age, without using them many times in the classroom, they find it very difficult.

Another issue is the role of writing in school education in general and mathematics in particular. Our school system demands demonstration of understanding only though written means; written examinations, reports etc etc. This is acceptable to a certain extent in high school where students are expected to have mastery over the skill of language and the skill of writing. But in Primary School, students are still struggling to master the skill of writing. They take a long time to write any piece of text. Hence a major portion of time in a class is spent in writing thus leaving very little time for understanding the concept.

For example, if a teacher is teaching word problems in addition, and expects students to write each problem in full, with all the steps, then only 2 or 3 problems can be done in a class. To understand word problems to figure out the process to be used (say addition or subtraction) students need to do many more problems. Ideally many problems should be discussed orally in class so that students get a hang of the concept. Students are never given this opportunity when teachers expect them to present each and every answer in writing with all the steps.

**Inadequate Understanding of the Child**

Teachers have very little understanding children's capacity for abstraction and the way they form concepts of mathematics. There is a considerable body of literature on this issue but it remains in the ivory tower of psychologists & universities. It never reaches teachers. Hence most teachers understand learn- ing as memorizing. There is very little understanding of what Understanding is. This ignorance affects Mathematics teaching to the maximum since Mathematics mostly consists of concepts which need understanding.

**Suggestions for the Classroom**

In the light of the above discussions, we give below some strategies to be adopted in the Mathematics classroom in Primary School to make learning accessible and enjoyable.

1. Relate concepts to learners' experience and previous knowledge Concepts are understood by relating them to our existing knowledge and experience.
2. Enable formation of mental images and patterns of concepts

   Concepts are stored in our mind in the form of images and patterns. Pro- vide opportunities for this.
3. Plan activities, if necessary, with 'designed' activity materials

   Many mathematical concepts may be related to experiences that a child may not have had. Design activities & materials which will help them internalize the related concepts.

4. Proper use of a Mathematics Lab

   The Math Lab has to be used like an Activity Centre rather than a mathematics museum.

5. Less emphasis on verbal explanations

   With activity materials, provide non-verbal means of learning and demonstrating

6. Move from the concrete to semi-concrete to abstract

   Children should be helped to construct concepts in their minds by starting with concrete experiences and then move on to semi concrete images and then to abstract ideas.

7. Encourage peer group interactions

   Concepts are also formed by discussing, defending and modifying our ideas. This has to be facilitated in a non-threatening and non-combative atmosphere.

8. Less emphasis on writing while trying to understand concepts
9. Learn information related to concepts only after understanding the concepts

   Introduce names and definitions after understanding.

10. Remember information by repeated use and not by memorizing

    Most mathematical concepts need not be memorized. Most mathematical facts can also be derived logically. Hence they need not be memorized. If they are used a number of times, remembering them will become easier.

11. Practice is different from drill.

    Drill is drudgery. Practice is doing a variety of problems to master all

    aspects of a problem. Drill is doing the same thing a number of times. Much of the drill-work given in schools is very boring.

12. Practice skills through exploration and interesting happy drills Mathematics has many areas where students can conduct open-ended explorations with surprise patterns and endings. These will make learning Mathematics 'fun' for children and at the same time provide them a lot of practice in skills.

**Suggestions for Structural Changes**

We also suggest the following changes at the policy level for any long term change in the school mathematics scenario.

1. Teacher Training

The content of the Teacher Education courses in Mathematics needs to be drastically revised. The unique character of Mathematics which sets it apart from other 'school subjects' has to be recognized .

2. Curriculum Transaction

   Since Mathematics is hierarchical in nature, it has to be constructed like a building where the work proceeds level by level from the lower to the higher. All the different kinds of work needed to complete a level have to be completed before moving to the next level.

A spiral coverage of curriculum is better than a linear coverage.

3. Refocus Mathematics Curriculum on processes

   Mathematics curriculum should be centred around mathematical processes like addition, subtraction, multiplication, division, comparison, tabulation etc. Numbers and shapes should form the medium through which the processes are understood.

   Concentrating on processes will also enable students to understand that Mathematics is also a language for expressing the processes

4. Design of Textbooks

   Present design of text books forces a linear coverage of the curriculum. The design has to be changed to facilitate a spiral coverage. In the lower classes, there is no need of a textbook since students do not have the ability to read and understand mathematical prose. In the Primary School we should shift to a system of a loose-leaf type Teachers' Handbook and Workbooks for students.